Wealth Taxation, Capital Gains Taxation and the Inequality–Mobility Trade-Off

Christophe Van Langenhove Jan Lorenz Jan Schulz *

Abstract

We compare wealth taxes and capital gains taxes in a random growth model with idiosyncratic investment risk. At equal tax revenue, wealth taxes generate higher wealth inequality than capital gains taxes, and higher wealth mobility wherever growth volatility is high enough. The mechanism is variance: wealth taxes shift the mean of post-tax wealth growth and raise its variance, while capital gains taxes compress the upper tail and reduce variance. Lower variance narrows the stationary distribution, reducing wealth inequality, but slows wealth rank changes, reducing wealth mobility, and at a baseline calibration the variance effect dominates. Policymakers who value both low wealth inequality and high wealth mobility therefore face a trade-off. The trade-off is robust to type and scale dependence in returns, hand-to-mouth agents, aggregate risk and tax progressivity, and full deductibility of capital losses below a two percent effective tax rate.

JEL classification: D31, H24, E21

Keywords: wealth taxation, capital gains taxation, wealth inequality, wealth mobility, random growth models, Monte Carlo simulations

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1 Introduction

Wealth taxes and capital gains taxes generate different combinations of wealth inequality and wealth mobility, even when raising the same tax revenue. For a given effective tax rate, wealth taxes produce higher wealth inequality than capital gains taxes, and higher wealth mobility wherever growth volatility is high enough. Policymakers who value both low wealth inequality and high wealth mobility therefore face a trade-off between inequality and mobility.

The mechanism operates through the cross-sectional variance of post-tax wealth growth rates. A wealth tax is a fixed fraction of opening wealth, so it takes a larger share of closing wealth after a bad draw than after a good one. It therefore lowers the mean of each agent's wealth growth and raises its variance. Capital gains taxes fall on positive gains alone, so they compress the upper tail and reduce variance. Lower variance narrows the stationary distribution, which reduces wealth inequality. For wealth mobility, two forces operate in opposite directions. Lower variance slows wealth rank changes. Lower wealth inequality means a smaller shock is enough to exit the top wealth decile. At the baseline calibration the variance effect dominates: agents under capital gains taxes are less likely to exit the top wealth decile.

We formalize this in a partial equilibrium random growth model with idiosyncratic investment risk and lump-sum redistribution. Agents hold undiversified portfolios and therefore face multiplicative shocks. We compare two tax instruments at equal effective tax rates: a wealth tax and a mark-to-market tax on unrealized capital gains. We characterize how each tax affects the variance of wealth growth, the stationary distribution, and the probability of exiting the top of that distribution. The trade-off between wealth inequality and wealth mobility holds at the baseline calibration, and is robust to type- and scale-dependent returns, hand-to-mouth agents, aggregate investment risk, autocorrelated shocks, weaker redistribution, other elasticities of intertemporal substitution and tax progressivity. Under an eighth extension, full deductibility of capital losses, the trade-off holds below a two percent effective tax rate.

Rising wealth inequality has renewed interest in wealth taxation (e.g., Guvenen et al., 2023, 2024; Saez & Zucman, 2019; Scheuer & Slemrod, 2021), and the case for a tax instrument is usually made on wealth inequality alone. But high wealth inequality with high wealth mobility is not the same as high wealth inequality with low wealth mobility (Van Langenhove, 2026), and the tax instrument that does better on one need not do better on the other. The choice of tax instrument matters only because returns differ across agents: where they are equal, a wealth tax and a capital income tax are equivalent at appropriate tax rates (Guvenen et al., 2023). Stochastic multiplicative dynamics break that equivalence, and this paper asks what the two tax instruments then do to wealth inequality and wealth mobility at once.

Related literature Our paper speaks to three strands of the literature on wealth inequality, taxation, and wealth mobility.

First, our paper joins a literature on wealth and capital gains taxation. Its oldest question is how a tax changes the risk an investor takes: Domar & Musgrave (1944) and Stiglitz (1969b) show the answer turns on loss offsets and risk aversion. Bastani & Waldenström (2020) and Scheuer & Slemrod (2021) survey the field, with behavioral responses estimated on Danish (Jakobsen et al., 2020), Swedish (Jakobsen et al., 2025), Swiss (Brülhart et al., 2022), Norwegian (Ring, 2025; Thoresen et al., 2022) and Belgian (Bastin et al., 2026) data, and reviewed for the UK by Advani & Tarrant (2021). Guvenen et al. (2023) show that wealth and capital income taxes are equivalent only under homogeneous returns, and a wider optimal-taxation literature adds entrepreneurial risk and macroeconomic effects (Boadway & Spiritus, 2025; Boar & Knowles, 2024; Boar & Midrigan, 2022; Gahvari & Micheletto, 2016; Gaillard & Wangner, 2023; Gerritsen et al., 2025; Guvenen et al., 2024; Kristjánsson, 2016).

Second, we build on random growth models of wealth inequality. Wold & Whittle (1957) derive a Pareto tail in a birth–death model of wealth accumulation and inheritance. Stiglitz (1969a) traces Champernowne's and Mandelbrot's Pareto conditions to random returns uncorrelated with wealth. Bouchaud & Mézard (2000) add mean-reverting redistribution to ensure stationarity, and Lorenz et al. (2013) show that it can raise aggregate growth by reducing variance drag. Benhabib et al. (2011, 2015) micro-found these dynamics with stochastic returns, and Gabaix et al. (2016) characterize their transition dynamics, showing that type or scale dependence in growth rates is needed to match movements in top shares. Jones (2015), Benhabib & Bisin (2018) and Schulz & Weber (2025) survey the field.

Third, a growing literature measures relative wealth mobility. Inter- and intra-generational wealth mobility is quantified for the United States (Fisher & Johnson, 2023; Fisher et al., 2016; Van Langenhove, 2026), the Nordic countries (Adermon et al., 2018; Audoly et al., 2024; Black et al., 2020; Boserup et al., 2017; Fagereng et al., 2021) and the United Kingdom (Gregg & Kanabar, 2023; Levell & Sturrock, 2023). On the theoretical side, Aiyagari-Bewley-Huggett frameworks are used to study the joint determinants of wealth inequality and wealth mobility (Atkeson & Irie, 2022; Benhabib et al., 2019, 2022; Fischer, 2019; Hubmer et al., 2024; Yang & Zhou, 2022). We integrate relative wealth mobility into the discussion on wealth taxation and capital gains taxation.

This paper measures how a tax instrument changes the variance of wealth growth at a given tax revenue. That variance sets the Pareto tail and the probability of exiting the top wealth decile. We state the condition under which the tax instrument that lowers wealth inequality also lowers wealth mobility. Comparing tax instruments at equal tax revenue goes back to Stiglitz (1969a), closed-form tails under random growth with redistribution to Gabaix et al. (2016) and the literature Benhabib & Bisin (2018) survey, and the risk-taking response to taxation to Domar & Musgrave (1944) and Stiglitz (1969b). None of them links a tax instrument's effect on variance to both wealth inequality and wealth mobility.

Roadmap Section 2 presents the model. Section 3 presents simulation results and explains the mechanism. Section 4 examines robustness. Section 5 concludes.

2 Model

2.1 Wealth dynamics

Time is discrete, indexed by $t \in \{0, 1, 2, \ldots\}$. A population of N infinitely-lived agents, indexed by $i \in \{1, \ldots, N\}$, have logarithmic utility over consumption and discount at rate $\rho > 0$:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_{i,t} \right] \tag{1}$$

where $\beta = 1/(1+\rho)$ is the discount factor. Each agent operates an individual investment technology.

Agent i's wealth earns an expected return $r > 0$ but faces idiosyncratic risk captured by $\varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, 1)$. The shocks $\varepsilon_{i,t}$ are independent across agents and time.

Consider first the case without taxation, then the case with taxes and redistribution. In the absence of taxation, wealth evolves as:

$$W_{i,t+1} = W_{i,t} \exp(\mu + \sigma \varepsilon_{i,t}) \tag{2}$$

where $\mu \equiv r - \rho - \frac{1}{2}\sigma^2$ is the expected log growth rate net of consumption and $\sigma > 0$ is the volatility of log growth. Under log utility, agents consume the constant fraction $1 - \beta \approx \rho$ of wealth each period, which the drift absorbs. The $-\frac{1}{2}\sigma^2$ term is a Jensen correction ensuring $\mathbb{E}[W_{i,t+1}/W_{i,t}] = \exp(r - \rho)$.

Log utility is a normalization. For any constant relative risk aversion θ, with i.i.d. returns and no labor income, the optimal rule is still $c_{i,t} = \omega W_{i,t}$, with $\omega = 1 - [\beta\, \mathbb{E}(R^{1-\theta})]^{1/\theta}$ for gross return R (Appendix G). A different elasticity of intertemporal substitution moves the drift and leaves the variance of log growth at σ^2.

With taxes and redistribution, wealth evolves as:

$$W_{i,t+1} = W_{i,t} \exp(\mu + \sigma \varepsilon_{i,t}) - \mathcal{T}_{i,t} + \mathcal{T}_t^{\text{redist}} \tag{3}$$

where $\mathcal{T}_{i,t}$ denotes taxes paid and $\mathcal{T}_t^{\text{redist}}$ denotes lump-sum transfers. The government redistributes tax revenue equally:

$$\mathcal{T}_t^{\text{redist}} = \frac{1}{N} \sum_{j=1}^{N} \mathcal{T}_{j,t} \tag{4}$$

which creates mean reversion in relative wealth (Benhabib et al., 2011; Gabaix et al., 2016). Without redistribution, relative wealth would be a martingale and no stationary distribution would exist. Fernholz & Fernholz (2014) prove that with uninsurable idiosyncratic investment risk and no redistribution, the wealthiest agent's time-averaged wealth share converges to one. Our contribution is to show that the effects of this stabilization on wealth inequality and wealth mobility differ between tax instruments.

The model isolates one channel: what each tax instrument does to the distribution of wealth growth at a given tax revenue. The model has no behavioral responses and no general equilibrium: portfolio composition, labor supply, entrepreneurial entry and asset prices do not react to the tax. The random growth literature makes the same abstraction (Benhabib & Bisin, 2018; Gabaix et al., 2016).

2.2 Taxation

Tax instruments We compare two tax instruments that differ in their base. For the capital gains tax, define pre-tax wealth as:

$$W_{i,t}^{\text{pre}} = W_{i,t} \exp(\mu + \sigma \varepsilon_{i,t}) \tag{5}$$

First, a wealth tax is proportional to wealth held at the start of the period and is paid out of closing wealth:

$$\mathcal{T}_{i,t}^{(W)} = \tau_W W_{i,t} \tag{6}$$

Second, an unrealized capital gains tax applies to positive capital gains, marked to market each period:

$$\mathcal{T}_{i,t}^{(U)} = \tau_U \max\{W_{i,t}^{\text{pre}} - W_{i,t}, 0\} \tag{7}$$

The unrealized capital gains tax is a mark-to-market tax: gains are taxed as they accrue, whether or not the asset is sold. Taxing gains on accrual has been proposed as a way to reach gains that are never realized (Saez & Zucman, 2019).[1]

Equal-ETR comparisons We compare the two tax instruments at equal effective tax rates (ETR), defined as the ratio of total tax revenue to total wealth:

$$\mathrm{ETR}_t \equiv \frac{\sum_{j=1}^{N} \mathcal{T}_{j,t}}{\sum_{j=1}^{N} W_{j,t}} \tag{8}$$

For the wealth tax, ETR $= \tau_W$ by construction, since the tax and the denominator share a base. For the capital gains tax the effective tax rate depends on the whole distribution of gains. We solve for the τ_U that hits a target tax rate. At a common effective tax rate the two tax instruments take the same expected amount, $\mathrm{ETR} \cdot W_{i,t}$, and leave the same expected growth factor, $\mathbb{E}[\exp(g)] - \mathrm{ETR}$, for pre-tax log growth g. The two differ only in the variance of post-tax growth.

Comparing at equal tax revenue ranks the two tax instruments without a welfare weight on either objective. That ranking is fair only where collecting the tax revenue costs the same under both. Administration, compliance and enforcement are costly, and avoidance rises with the statutory tax rate (Slemrod & Yitzhaki, 2002). We assume those costs do not differ between the two tax instruments.

An equal effective tax rate is not an equal statutory tax rate. Because its base is one year's gains rather than a stock of wealth, the capital gains tax needs a higher statutory tax rate to raise the same tax revenue: at a five percent effective tax rate it runs at roughly thirty-eight. The model has no avoidance, so a real capital gains tax at thirty-eight percent would probably raise less than the model assumes.

3 Simulation results

3.1 Baseline calibration

We simulate the model with $N = 10{,}000$ agents over $T = 200$ time periods in partial equilibrium, each starting from initial wealth $W_0 = 1$. The expected return is $r = 0.08$, the discount rate $\rho = 0.06$ and the volatility parameter $\sigma = 0.30$. Together these give a log growth rate of

[1] We also explored a realization-based variant, in which the tax was triggered once cumulative gains relative to a cost basis crossed a fixed threshold. We do not report it. With no portfolio to rebalance, crossing the threshold is mechanical rather than a decision, so the variant is a periodic tax on unrealized gains under another name.

$\mu = r - \rho - \frac{1}{2}\sigma^2 = -0.025$. The return is in the range Jordà et al. (2019) measure for long-run real returns on risky assets.

The baseline sets $\sigma = 0.30$. An agent here holds one undiversified position and no labor income. Its counterpart in the data is directly held private equity, which Bach et al. (2020) put at 51% a year for the median holder and about 40% at the top.[2] The baseline is below both, which is the conservative direction: the wealth mobility result needs volatility above a threshold.

We vary the effective tax rate ETR from 0% to 5%.[3] At the baseline expected return of eight percent, that range is equivalent to taxing capital income at between 12.5% and 62.5%. Three countries still levy a wealth tax: Norway, Switzerland and Spain. Norway's statutory tax rate has run between 0.85% and 1.1% (Ring, 2025), and effective tax rates are lower again because the tax applies only above an exemption threshold: Mas-Montserrat et al. (2025) measure the Spanish wealth tax at 0.30% of reported wealth in Catalonia in 2011. No country levies a mark-to-market tax on unrealized capital gains at all.

3.2 Baseline results

We obtain two results (Figure 1). First, at a given effective tax rate the wealth tax generates higher wealth inequality than the capital gains tax, but also higher wealth mobility. Second, both gaps widen as the effective tax rate rises. Together they are a trade-off: reducing wealth inequality comes at the cost of lower wealth mobility.

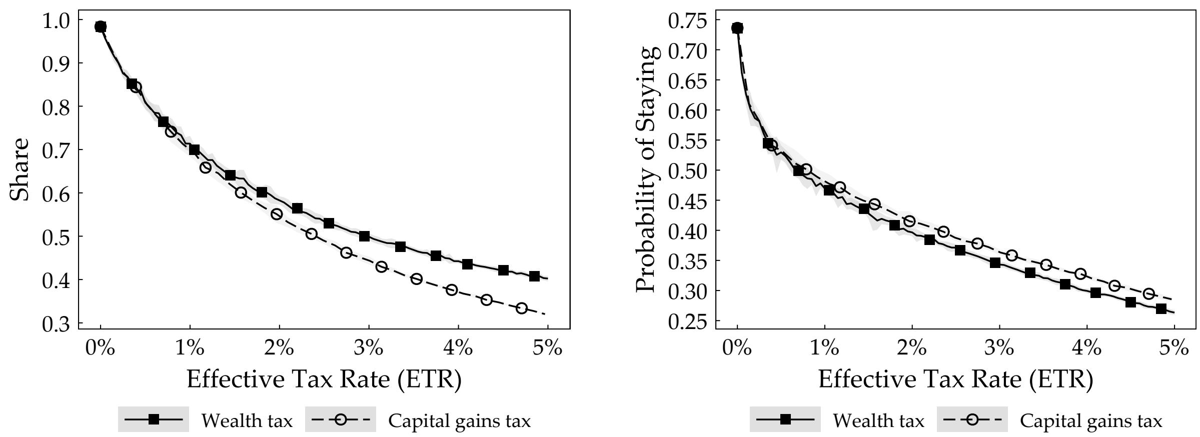
Both results survive the choice of statistic, horizon and stopping time (Appendix A). They hold for wealth shares at the bottom and middle of the distribution as well as at the top, at exit horizons from five periods to fifty rather than twenty, and at $t = 100$ rather than the $t = 200$ the paper reports, by which point the distribution has settled. Section 4 shows the robustness of the baseline results to alternative model formulations.

The trade-off is already visible at a one percent effective tax rate. At $\sigma = 0.30$ and a one percent effective tax rate the wealth tax leaves the top wealth decile with 2.0 points more of wealth than the capital gains tax, and a 1.7-point higher probability of exiting it. Both gaps are statistically significant, and both widen with the effective tax rate: at five percent they are 8.2 and 2.2 points.

[2] Diversified net wealth is far less volatile, 6% for the ninth decile and 27.5% for the top 0.01 percent (Bach et al., 2020) and 8.6% value-weighted for Norway (Fagereng et al., 2020), but each of those series mixes housing, pensions and public equity, which is not what an agent here holds.

[3] For the wealth tax, ETR $= \tau_W$ by construction. For the unrealized capital gains tax, an ETR of 5% corresponds to $\tau_U \approx 0.38$. A reported ETR corresponds to the ETR in the stationary distribution of the model.

Figure 1: The two tax instruments compared, and the inequality–mobility trade-off.



Notes: The left panel shows the top 10% wealth share. The right panel shows the probability of staying in the top 10% after 20 periods. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation (barely visible due to low cross-run variance).

3.3 The variance mechanism

The inequality–mobility trade-off arises because the two taxes move the variance of wealth growth in opposite directions. Figure 2 illustrates.

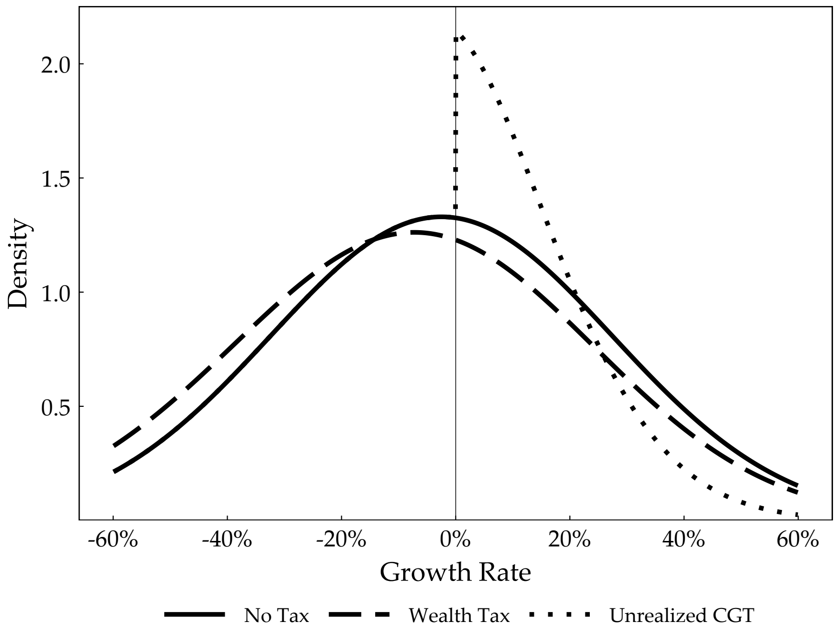
Post-tax wealth growth rates Define the pre-tax log growth rate as $g_t = \mu + \sigma\varepsilon_t$, so that $W_t^{\text{pre}} = W_t \exp(g_t)$. We define the post-tax log growth rate $\tilde{g}_t$ by $W_{t+1} = W_t \exp(\tilde{g}_t)$, abstracting from redistribution; post-tax volatility is written $\tilde{\sigma}$. Each tax implies a different growth rate $\tilde{g}_t$. First, the wealth tax falls on wealth held at the start of the period, so that $W_{t+1} = W_t[\exp(g_t) - \tau_W]$:

$$\tilde{g}_t^W = \log(\exp(g_t) - \tau_W) \tag{9}$$

The wealth tax takes $\tau_W W_t$, a fixed fraction of opening wealth that does not vary with the shock ε_t. After a bad draw it is therefore a larger share of closing wealth than after a good one, so the tax costs most in the states where wealth is already low. Equation (9) is therefore concave with slope above one everywhere, and it lowers the mean growth rate while widening the spread of growth rates. At the baseline $\tilde{\sigma}_W^2$ exceeds σ^2 by twelve percent. Second, under the unrealized capital gains tax, taxes apply to positive level gains $W_t(\exp(g_t) - 1)$. Post-tax wealth satisfies $W_{t+1} = W_t[\exp(g_t) - \tau_U \max\{\exp(g_t) - 1, 0\}]$. The post-tax growth rate is then:

$$\tilde{g}_t^U = \begin{cases} \log((1-\tau_U)\exp(g_t) + \tau_U) & \text{if } g_t > 0 \\ g_t & \text{if } g_t \leq 0 \end{cases} \tag{10}$$

Figure 2: Pre-tax and post-tax wealth growth distributions.



Notes: Densities of post-tax log wealth growth, with the two tax instruments set to a common effective tax rate of 5%, the top of the range in Figure 1. The wealth tax (dashed) moves the distribution left and widens it: the tax is a fixed fraction of opening wealth, so it takes a larger share of closing wealth after a bad draw than after a good one. The unrealized capital gains tax (dotted) coincides with the no-tax density over negative growth and compresses positive growth toward zero, thinning the right tail. The vertical rule marks zero growth, where the capital gains base starts. Variance rises by 12% under the wealth tax and falls by 28% under the capital gains tax. Parameters: $\mu = -0.025$, $\sigma = 0.30$, and at that effective tax rate $\tau_W = 0.05$ and $\tau_U = 0.38$.

which shows that positive growth rates are compressed by a transformation whose slope is below one, reducing variance. The post-tax variances therefore satisfy (see Appendix B.1):

$$\tilde{\sigma}_W^2 > \tilde{\sigma}_U^2 \tag{11}$$

Wealth inequality & wealth mobility In random growth models with redistribution, the stationary distribution has a Pareto right tail with exponent ζ characterized by the Kesten (1973) condition $\mathbb{E}[\exp(\zeta\tilde{g}_t)] = 1$. Writing $\gamma = -\mathbb{E}[\tilde{g}_t]$ for the strength of mean reversion, this condition solves in closed form (Benhabib & Bisin, 2018; Gabaix et al., 2016; Appendix B.2):

$$\zeta = \frac{2\gamma}{\tilde{\sigma}^2} \tag{12}$$

Lower variance implies higher ζ: a thinner tail and lower top wealth shares. The variance ordering therefore determines the wealth inequality ordering:

$$\zeta_W < \zeta_U \tag{13}$$

We measure wealth mobility as the probability that an agent in the top wealth decile has fallen below the ninetieth percentile h periods later. Under the Pareto tail, the log distance d from that agent down to the threshold is exponential with rate ζ. Over h periods the agent's own log wealth drifts by $-\gamma h$ and diffuses with variance $h\tilde{\sigma}^2$. The agent leaves the decile if its log wealth falls by more than d over those h periods. Averaging over where the agent sits inside the decile gives a closed form (Appendix B.3):

$$P(\text{exit at } h) = 2\,\Phi\left(\frac{\gamma\sqrt{h}}{\tilde{\sigma}}\right) - 1 \tag{14}$$

Equation (14) depends on the tax instrument only through $\gamma/\tilde{\sigma}$, and γ has two parts. The tax takes a deterministic amount, $\lambda \equiv -\log \mathbb{E}[\exp(\tilde{g}_t)] = -\log(\exp(r-\rho) - \text{ETR})$. Multiplicative shocks add a variance drag of $\frac{1}{2}\tilde{\sigma}^2$, which is the gap Jensen's inequality opens between mean reversion in logs and in levels. So $\gamma = \lambda + \frac{1}{2}\tilde{\sigma}^2$, and Equation (14) becomes:

$$P(\text{exit at } h) = 2\,\Phi\left(\sqrt{h}\left[\frac{\lambda}{\tilde{\sigma}} + \frac{\tilde{\sigma}}{2}\right]\right) - 1 \tag{15}$$

The first term, $\lambda/\tilde{\sigma}$, falls in $\tilde{\sigma}$: a thicker tail spreads the top wealth decile out, so an agent in it sits further above the threshold and has further to fall. The second, $\tilde{\sigma}/2$, rises: bigger shocks make a fall of any given size more likely. Their sum is minimized at $\tilde{\sigma} = \sqrt{2\lambda}$ and rises above it.

At a matched effective tax rate each tax instrument takes $\text{ETR} \cdot W_{i,t}$ in expectation from the same base, so λ is the same under both. The two therefore differ only in $\tilde{\sigma}$. Above $\tilde{\sigma} = \sqrt{2\lambda}$ wealth mobility rises in $\tilde{\sigma}$, so the tax instrument that leaves more variance is the more mobile one, and it is enough that the compressing tax instrument clears that point:

$$\tilde{\sigma}_U^2 > 2\lambda \tag{16}$$

Hence:

$$P(\text{exit})_W > P(\text{exit})_U \tag{17}$$

The condition is sufficient, not necessary: where it holds the trade-off holds, and where it fails nothing follows. It applies only at effective tax rates above two percent, where the drag is positive. The bound it puts on volatility is loose: at a five percent effective tax rate the condition requires $\sigma > 0.29$, while simulation places the boundary between 0.175 and 0.20.[4]

[4] Combining Equation (12) with $\gamma = \lambda + \frac{1}{2}\tilde{\sigma}^2$ gives $\zeta = 1 + 2\lambda/\tilde{\sigma}^2$, so Equation (16) is equivalent to $\zeta_U < 2$, the infinite-variance Pareto case. The two forms disagree at the baseline: the closed form puts ζ_U at 1.93 and the

3.4 The growth volatility threshold

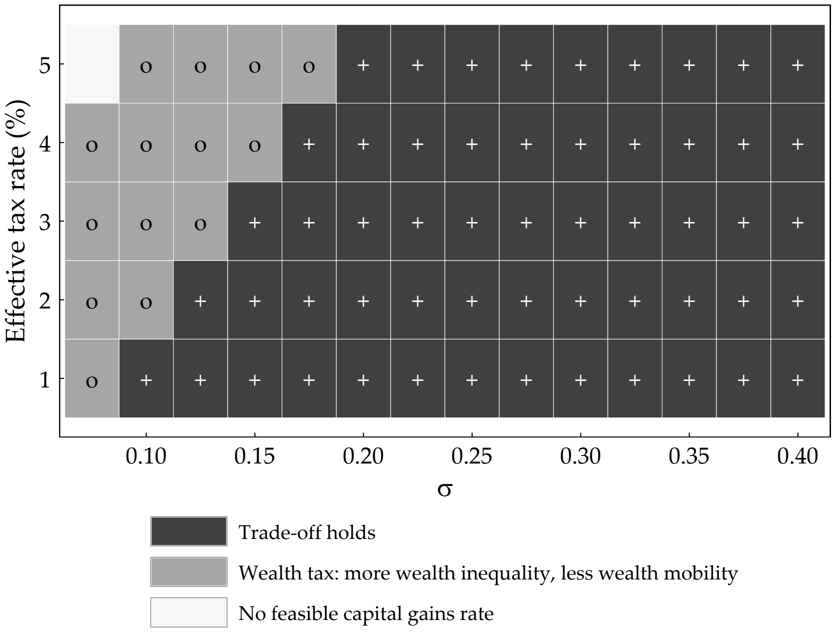
Does the trade-off hold at every volatility and effective tax rate? The wealth inequality ordering does, and the wealth mobility ordering does not.

First, the wealth tax produces more wealth inequality than the capital gains tax at every volatility and effective tax rate we can compare. The wealth inequality gap is positive at each of the sixty-nine combinations we can compare (Figure 3), and it widens with the effective tax rate at every σ (Appendix A.3). Two are not statistically significant, both at a one percent effective tax rate and at the two highest volatilities, where the gap is 0.1 and 0.5 points.

Second, the wealth tax produces more wealth mobility only above a growth volatility threshold. That threshold rises with the effective tax rate, from $\sigma \approx 0.10$ at one percent to $\sigma \approx 0.20$ at five. The trade-off holds above the threshold, at fifty-five of the sixty-nine combinations. Below it the capital gains tax is both less unequal and more mobile, so the trade-off does not arise.

exact Kesten root at 2.03, either side of two. We therefore check the condition on $\tilde{\sigma}^2$, where both sides are measured the same way. Appendix B.3 gives both calculations, and the trade-off holds at the baseline in simulation on either reading.

Figure 3: The region of the inequality–mobility trade-off.



Notes: Fourteen volatilities by five effective tax rates. Each cell runs both tax instruments at the stated effective tax rate, 25 simulations each, $N = 10{,}000$, $T = 200$, σ from 0.075 to 0.40 in steps of 0.025, $r = 0.08$, $\rho = 0.06$. Every simulated volatility is one column of equal width. The two tax instruments' achieved effective tax rates never differ by more than 9×10^{-5}, so each cell compares equal tax revenue. A cell marked $+$ is one where the wealth tax produces both more wealth inequality and more wealth mobility, which is the paper's claim. A cell marked with an o is one where it produces more wealth inequality and less wealth mobility, so that the capital gains tax dominates on both counts. A blank cell is one where a capital gains tax cannot raise the target tax revenue at any statutory tax rate below one hundred percent. The shading repeats the same three states, so the figure reads in black and white. The panels behind this one, reporting the two gaps separately, are in Appendix A.3.

4 Robustness of the trade-off

Section 3.2 reports two baseline results. First, at a common effective tax rate the wealth tax produces more wealth inequality than the capital gains tax. Second, it produces more wealth mobility. We test both against eight extensions of the model, one to a paragraph below, each with its own appendix.

Both results are robust to seven of the eight extensions. Table 1 sets four of them side by side at a common 5% effective tax rate. The wealth tax leaves the top wealth decile with between 2.8 and 9.2 percentage points more of total wealth, and with a probability of staying in it between 2.2 and 4.5 points lower. The eighth extension is full deductibility of capital losses. The wealth inequality result is robust to it, and the wealth mobility result reverses above a two percent effective tax rate.

Type and scale dependence Returns may vary structurally across agents, and we consider two forms of that variation (in line with e.g., Gabaix et al., 2016; Jones & Kim,

Table 1: Magnitudes across extensions, at a 5% effective tax rate.

	Top wealth decile share		P(stay in top wealth decile)	
	Wealth tax	Capital gains tax	Wealth tax	Capital gains tax
Baseline	0.40	0.32	0.26	0.29
Type dependence	0.42	0.33	0.28	0.30
Scale dependence	0.45	0.36	0.31	0.33
Hand-to-mouth, $\psi = 0.9$	0.44	0.36	0.29	0.31
Hand-to-mouth, $\psi = 0.7$	0.54	0.45	0.34	0.38
Aggregate risk, $\kappa = 0.9$	0.38	0.31	0.27	0.29
Aggregate risk, $\kappa = 0.8$	0.36	0.30	0.27	0.30
Progressive, $\chi = 0.5$	0.31	0.25	0.17	0.21
Progressive, $\chi = 1$	0.25	0.22	0.12	0.16

Notes: Each row is that extension's combination nearest a 5% effective tax rate, 25 simulations per combination. The capital gains tax is the unrealized capital gains tax. Higher is more unequal on the left and less mobile on the right. The paper's claim is a higher top wealth decile share under the wealth tax together with a lower probability of staying in it, which holds in every row. The two tax instruments' achieved effective tax rates differ by less than a tenth of a percentage point anywhere in the table. The aggregate-risk rows were re-run on a finer grid of statutory tax rates, because the original grid was too coarse to place both tax instruments at one effective tax rate. Rows at the baseline parameters (hand-to-mouth at $\psi = 1$, aggregate risk at $\kappa = 1$, progressive taxation at $\chi = 0$) are omitted. Four extensions are absent from the table. The elasticity of intertemporal substitution, the redistribution rule and autocorrelated shocks vary preferences, the transfer rule and the shock process rather than a share of the population or of variance, and each is reported at the same effective tax rate in its own table in Appendices G, H and I. Capital loss offsets cannot reach a 5% effective tax rate at all. Gaps quoted in the text are computed before rounding, so they need not equal the difference of the entries printed here.

2018). Type dependence lets agents transition between $K = 5$ types with expected returns $r_k \in \{0.06, 0.07, 0.08, 0.09, 0.10\}$ at persistence $\pi = 0.99$. Scale dependence raises expected returns linearly across wealth deciles, from 0.06 for the bottom decile to 0.10 for the top wealth decile, centered on the baseline $r = 0.08$. Both raise wealth inequality and lower wealth mobility relative to baseline. The variance ordering is unchanged under either, so the capital gains tax still compresses variance more than the wealth tax and the trade-off holds. Scale dependence in volatility rather than in returns is the ramp of Appendix B.3, which does not (Appendix C).

Hand-to-mouth agents Not all agents hold wealth. We introduce hand-to-mouth agents who hold zero wealth (in line with e.g., Galí et al., 2007; Kaplan et al., 2018). A fraction $\psi \in \{1.0, 0.9, 0.7\}$ are entrepreneurs following baseline dynamics, and the remainder are hand-to-mouth. The baseline is $\psi = 1$. Only entrepreneurs pay taxes, and tax revenue is shared across all agents, so a lower ψ returns less of it to the agents that paid it and weakens the mean reversion redistribution creates. Wealth inequality rises and wealth mobility falls as ψ declines. The variance mechanism among entrepreneurs is untouched, so the trade-off holds (Appendix D).

Aggregate risk The baseline model assumes purely idiosyncratic risk. We add aggregate shocks that affect all agents (in line with e.g., Krusell & Smith, 1998; Hubmer et al., 2021). Let $\kappa \in \{1.0, 0.9, 0.8\}$ denote the idiosyncratic share of total variance, with the baseline at $\kappa = 1$. At $\kappa = 0$ all agents move together and wealth inequality vanishes. For $\kappa > 0$ only idiosyncratic variance drives relative wealth dynamics, and the capital gains tax compresses that component more than the wealth tax. The trade-off therefore holds wherever the simulations resolve it (Appendix E).

Tax progressivity Real-world tax proposals often include exemption thresholds (Drometer et al., 2018; Saez & Zucman, 2019; Scheuer & Slemrod, 2021; Wolff, 2020). We tax only wealth, or positive capital gains, above $\chi \in \{0, 0.5, 1\}$ times its mean, where $\chi = 0$ is the baseline and taxes from the first unit. A higher threshold requires a higher statutory tax rate for equal tax revenue. A tax above an exemption still charges a fraction of opening wealth fixed before the return arrives, so the map of Equation (9) applies at the agent's effective tax rate $\tau_W(1 - \chi \bar{W}_t / W_{i,t})$. The variance ordering holds within the taxed population and the trade-off holds wherever the simulations resolve it (Appendix F). Of the extensions in Table 1, progressive taxation compresses wealth inequality most, and it is where the wealth inequality gap is narrowest and the wealth mobility gap widest.

Elasticity of intertemporal substitution The elasticity of intertemporal substitution cannot move the variance ordering. It sets how much of wealth an agent consumes, and so the drift, and it leaves the variance of log growth at σ^2, which is the only thing either tax acts on. Both orderings hold from an elasticity of 2 down to 0.4, against the baseline's log utility at 1. Below 0.4 the agent's problem has no interior solution at our volatility under the common consumption rule, and below about 0.6 under a rule re-optimised against each tax instrument's post-tax return. Lower elasticities are untested because the model has no solution there (Appendix G).

Redistribution rule How tax revenue is returned moves the level of wealth inequality and not the ordering between tax instruments. We return a fraction $\alpha \in \{1, 0.75, 0.5, 0.25\}$ of tax revenue as an equal transfer and the rest in proportion to wealth. At $\alpha = 1$ every agent receives the same amount, and as α falls the transfer shifts towards the wealthy. The top wealth decile's share rises from 0.4 to 0.7 as α falls. The ordering does not move, because the deterministic drag λ is common to both tax instruments. Both gaps are wider at every α below one than at the baseline (Appendix H).

Autocorrelated shocks Both orderings hold when the return shocks are autocorrelated. We replace $\varepsilon_{i,t}$ with $\varepsilon_{i,t} = \rho_\varepsilon \varepsilon_{i,t-1} + \sqrt{1-\rho_\varepsilon^2}\,\nu_{i,t}$, where $\nu_{i,t}$ is standard normal, which holds the unconditional variance at one so that σ keeps its meaning. The wealth tax produces more wealth inequality and more wealth mobility at every ρ_ε tested. Persistence thickens the tail: the Hill estimate falls from 1.83 at $\rho_\varepsilon = 0$ to 0.94 at $\rho_\varepsilon = 0.75$. Below one the distribution has no finite mean. Kesten's theorem assumes independent shocks, so we read both orderings from simulation rather than from Equation (12) (Appendix I).

Capital loss offsets An unlimited offset for capital losses reverses the wealth mobility ordering. The baseline allows no offset, and we add one with no cap and no expiry. At a one percent effective tax rate the wealth tax is still the more mobile tax instrument. At two the gap is within sampling error, and at three and four percent the capital gains tax is both less unequal and more mobile. The reversal occurs only at tax rates no government levies: an effective tax rate of three percent requires a statutory tax rate of 70% on net capital gains, and no statutory tax rate delivers an effective tax rate above 4.1% (Appendix J).

5 Conclusion

At equal tax revenue, a government choosing between a wealth tax and a capital gains tax faces a trade-off between wealth inequality and wealth mobility. Wealth taxes lower the mean of wealth growth and raise its variance. Capital gains taxes compress the upper tail and reduce variance. Lower variance thins the tail, which lowers wealth inequality at every combination the simulations resolve. It also means smaller shocks, so agents change wealth ranks less often, which lowers wealth mobility wherever growth volatility is high enough.

The analysis has three limitations. First, the wealth mobility ordering holds only above a volatility of about 0.10 at a one percent effective tax rate, rising to about 0.20 at five. Second, an unlimited offset for capital losses reverses that ordering above an effective tax rate of about two percent, well above the effective tax rates wealth taxes are observed to collect. Third, the model has no behavioral responses and no general equilibrium, and it taxes one asset. Portfolio choice matters most, and it works against the result. A capital gains tax with no loss offset lowers demand for the risky asset (Domar & Musgrave, 1944; Stiglitz, 1969b), so agents free to rebalance would hold less volatile portfolios. That moves volatility toward the threshold below which the trade-off disappears.

Where the trade-off holds, both objectives move with the variance of post-tax wealth growth. No tax instrument lowers wealth inequality and raises wealth mobility at once. At a five percent effective tax rate the two tax instruments differ by 8.2 points of the top wealth decile's share and 2.2 points of the probability of staying in it. Ranking the two therefore requires a social welfare function that prices wealth inequality against wealth mobility, and this paper does not provide one.

Declarations of interest: none.

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A Baseline checks

Both orderings hold at every point of the wealth distribution the simulations resolve, not only at the top. At the bottom of the plotted tax rate range the top one percent panels do not resolve, and two of them run the other way inside one and a half standard errors. The capital gains tax generates lower wealth inequality than the wealth tax at equal tax revenue: higher wealth shares at the bottom and middle, lower wealth shares at the top (Figure 4). The probability of staying in a given wealth group is higher under the capital gains tax, which is lower wealth mobility (Figure 5).

This appendix checks the reported statistics rather than the model, and reads the two gaps behind the region map of Section 3.4. Each extension of Section 4 has an appendix of its own. A gap at a matched effective tax rate is read by linear interpolation of each tax instrument's series between its adjacent combinations, and its standard error from the two series' interpolated cross-run dispersion. A combination at exactly one percent therefore need not be simulated to be compared. Where an extension's appendix compares the two tax instruments at matched effective tax rates, it reports every gap that falls within sampling error.

A.1 Convergence

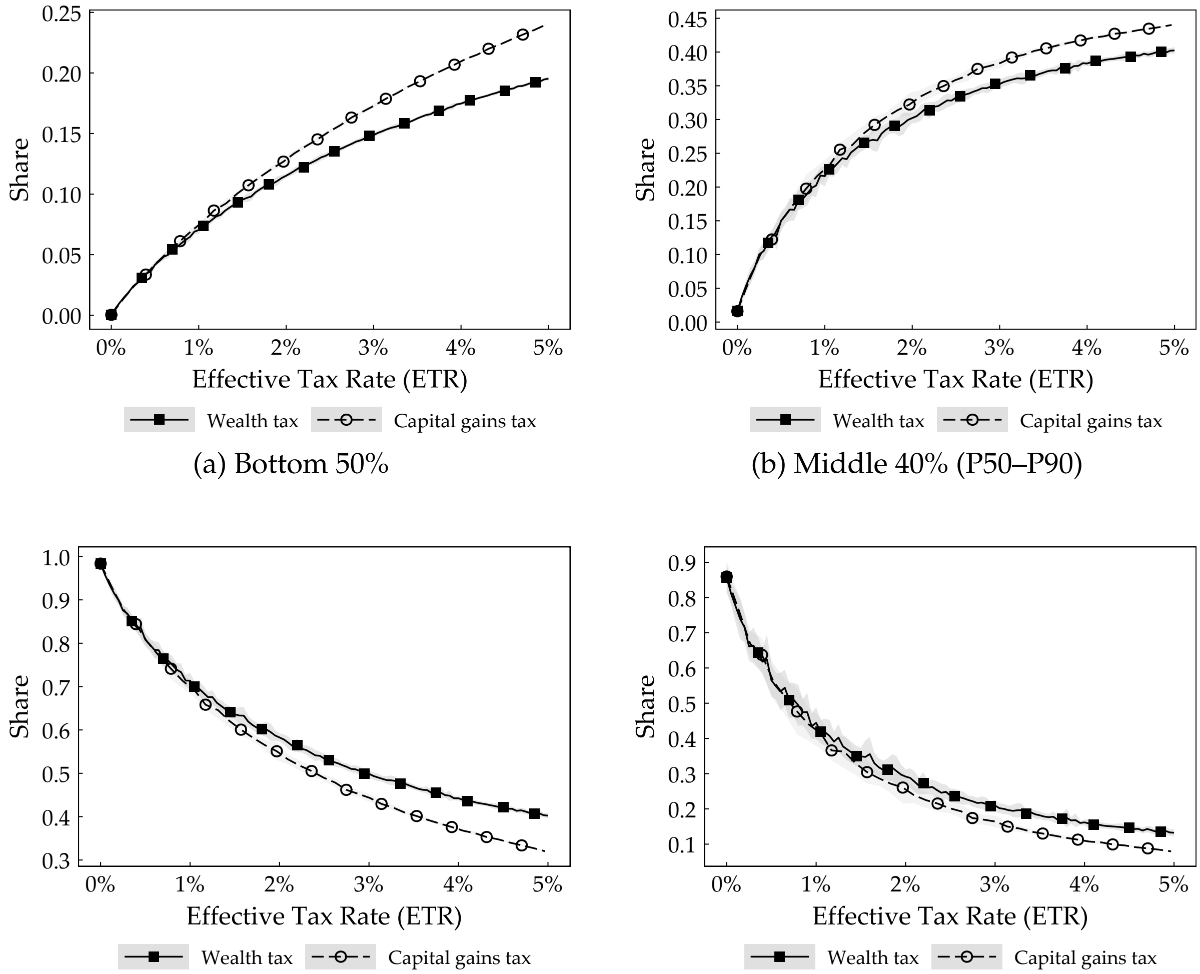
The distribution has settled well before the averaging window opens. Nothing moves by more than two thousandths over the second half of the run, and the two tax instruments are equally settled. The averaging window is not doing work that a longer run would undo. The statistics in Section 3 are averaged over the last hundred periods of a two-hundred-period run. Table 2 compares the top wealth decile's share and the Gini coefficient at $t = 100$ and at $t = 200$ under both tax instruments, at a common effective tax rate of 5%.

A.2 The wealth mobility horizon

The ordering does not depend on the horizon. At the baseline calibration the wealth tax is the more mobile tax instrument at every horizon tested, from five periods to fifty (Table 3). The paper measures wealth mobility as the probability of remaining in the top wealth decile after twenty periods, and that horizon is a choice. Equation (43) says it should not matter: exit depends on the horizon only through $\sqrt{h}$, which is monotone and common to both tax instruments, so the curves should not cross.

The horizon bears on the sign only at the boundary of Section 3.4, where the gap is small enough that sampling error decides it. There, two of the sixty-nine combinations disagree

Figure 4: Wealth inequality across the distribution under the two tax instruments.



(a) Bottom 50% (b) Middle 40% (P50–P90)

(c) Top 10% (d) Top 1%

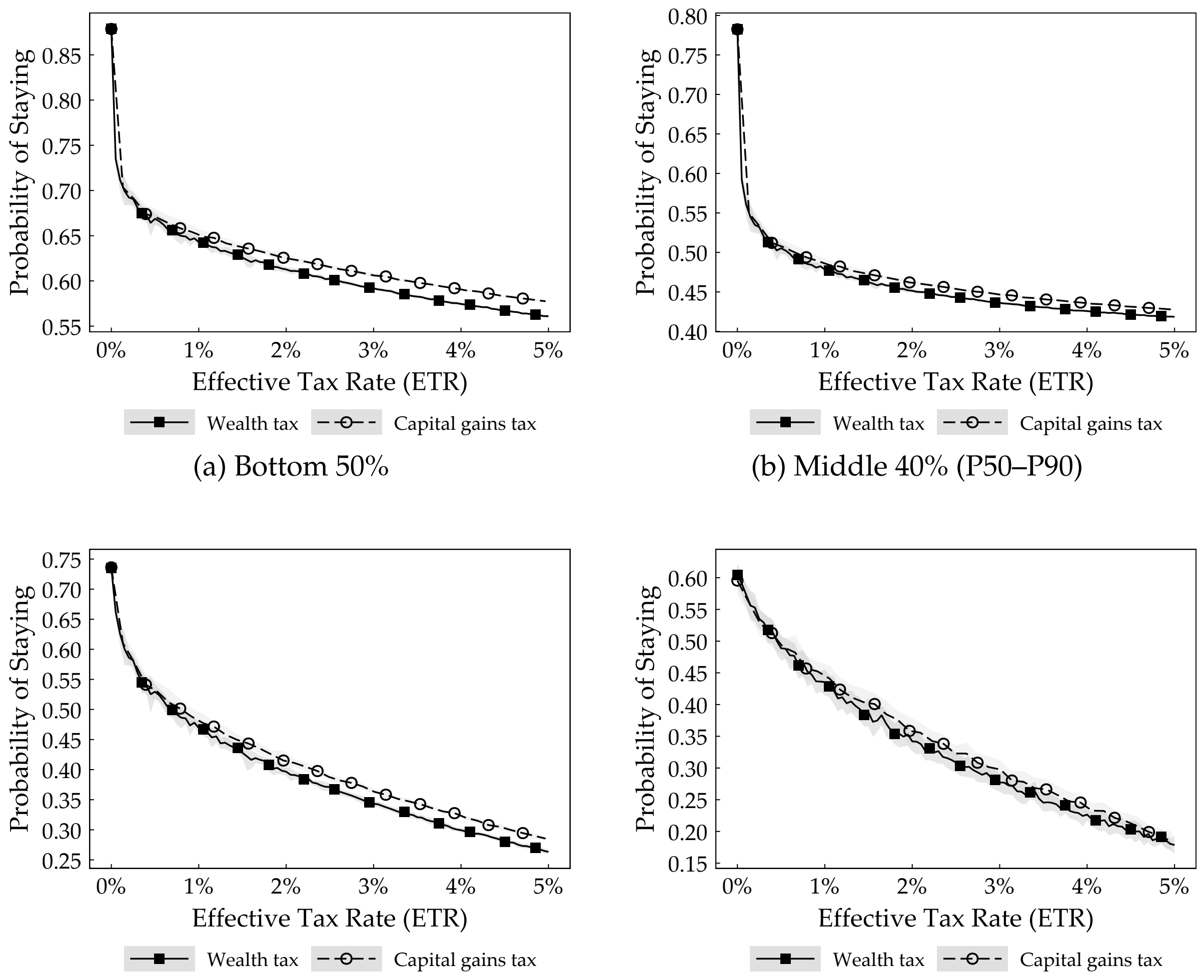
Notes: Each panel shows the wealth share held by the indicated group. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation, barely visible where cross-run variance is low and wide for the top 1%.

across horizons of ten, twenty and thirty. The baseline gap of Table 3 narrows at $h = 50$, which is what the $\sqrt{h}$ scaling implies once both probabilities are small. The gap does not close: at that horizon it is still more than nine standard errors from zero by its own cross-run dispersion. By the widest dispersion in the table it is more than five, measured as the other appendices measure their gaps.

A.3 The size of the two gaps

The wealth inequality gap is positive everywhere and the wealth mobility gap is not. The wealth inequality gap widens in the effective tax rate at every volatility. Its shape in volatility

Figure 5: Wealth mobility across the distribution under the two tax instruments.



(a) Bottom 50%

(b) Middle 40% (P50–P90)

(c) Top 10%

(d) Top 1%

Notes: Each panel shows the probability of staying in the indicated wealth group after 20 periods. Higher values indicate lower wealth mobility. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation, barely visible where cross-run variance is low and wide for the top 1%.

depends on the tax rate: at one and two percent it is smallest in the upper part of the range simulated, and at three percent and above at the bottom. The wealth mobility gap changes sign along the boundary described in Section 3.4. Figure 3 reduces each combination to which of three states it is in. Figure 6 shows the two gaps behind the state, so the size of each can be read as well as its sign.

The low-volatility corner is unreachable, not merely unfavorable. A capital gains tax cannot raise more than $\mathbb{E}[(\exp(g) - 1)^{+}]$ per unit of wealth without a statutory tax rate above one hundred percent, and that base rises with volatility. At the bottom of the range simulated the

Table 2: Convergence between $t = 100$ and $t = 200$.

	$t = 100$	$t = 200$	Change
Top 10% share, wealth tax	0.404	0.403	-0.001
Top 10% share, capital gains tax	0.322	0.322	$+0.000$
Gini, wealth tax	0.487	0.485	-0.002
Gini, capital gains tax	0.400	0.400	-0.000

Notes: Averages over 25 independent simulations, in a run separate from Table 1's. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Both tax instruments at an effective tax rate of 5%, which is $\tau_W = 0.05$ and $\tau_U = 0.38$.

Table 3: The wealth mobility ordering across horizons.

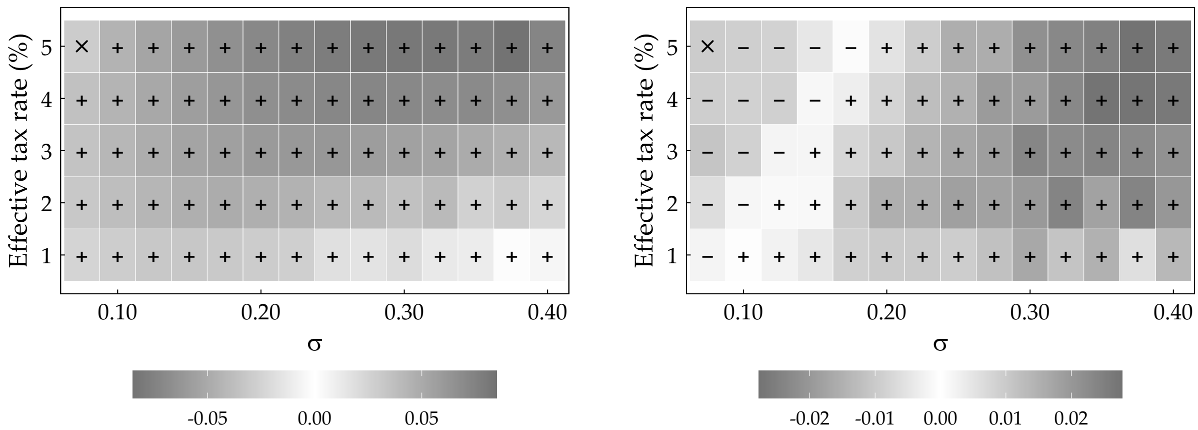
Horizon h	Wealth tax	Capital gains tax	Gap
5	0.560	0.579	-0.019
10	0.414	0.436	-0.022
20	0.263	0.283	-0.021
50	0.128	0.136	-0.008

Notes: Probability of remaining in the top wealth decile after h periods, averaged over 25 independent simulations. A negative gap means the wealth tax is the more mobile tax instrument, which is the paper's claim. Cross-run standard deviations range from 0.003 to 0.005. Parameters as in Table 2. Those rows are a separate run rather than a slice of the simulations behind Table 1, so the $h = 20$ row sits within sampling error of that table's baseline row rather than reproducing it.

shortfall is absolute: at $\sigma = 0.075$ the tax instrument cannot raise five percent of wealth at all. The same arithmetic makes the statutory tax rates required near the boundary severe, 98% at $\sigma = 0.10$ and a five percent effective tax rate.

The two wealth inequality gaps inside two standard errors of zero both sit at one percent, at $\sigma = 0.375$ and $\sigma = 0.40$, at 0.4 and 1.3 standard errors. Every other is significant. The gaps near the boundary are small relative to sampling error, and only at four and five percent does the first positive combination clear two standard errors. The boundary should therefore be read as a band and not a line. Away from that band the sign of the wealth mobility gap is robust to the horizon. Across horizons of ten, twenty and thirty periods the three agree in sign at sixty-seven of the sixty-nine combinations, and both exceptions have gaps under 0.002.

Figure 6: The wealth inequality and wealth mobility gaps at every volatility and effective tax rate simulated.



Notes: Left, the top wealth decile's wealth share under the wealth tax minus the same under the capital gains tax. Right, the top wealth decile exit rate under the wealth tax minus the same under the capital gains tax. Darker cells are larger gaps, the scale is centered on zero in both panels, and each cell carries the sign of its gap as a bold $+$ or $-$, because shade alone cannot separate a large positive gap from a large negative one. A positive exit-rate gap means the wealth tax is the more mobile tax instrument, the opposite sign to the stay gaps of Tables 3 and 7. The cross marks the combination where a capital gains tax cannot raise the target tax revenue at any statutory tax rate below one hundred percent. Each combination is 25 simulations per tax instrument, σ from 0.075 to 0.40 in steps of 0.025, $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$. Every simulated volatility is one column of equal width.

B Analytical results

Three results of Section 3 are derived here. Appendix B.1 establishes the variance ordering across tax instruments, and Appendix B.2 turns that ordering into a wealth inequality ordering through the Pareto tail exponent. Appendix B.3 derives the condition under which it orders wealth mobility as well.

B.1 Variance ordering

We show that post-tax growth variance satisfies $\tilde{\sigma}_W^2 > \tilde{\sigma}_U^2$. Intuitively, a wealth tax shifts the growth distribution leftward and, because it is assessed on opening wealth, widens it, while a capital gains tax compresses the right tail, reducing variance.

Recall from Section 3 that the (pre-tax) log growth rate is $g_t = \mu + \sigma\varepsilon_t$, where $\varepsilon \sim \mathcal{N}(0,1)$. Consistent with discrete-time geometric Brownian motion, pre-tax wealth evolves as:

$$W_{i,t}^{\text{pre}} = W_{i,t}\exp(g_{i,t}) \tag{18}$$

We define the post-tax log growth rate $\tilde{g}_{i,t}$ by $W_{i,t+1} = W_{i,t}\exp(\tilde{g}_{i,t})$, abstracting from redistribution (which is common across tax instruments in this variance comparison), and write $\tilde{\sigma}^2 \equiv \text{Var}(\tilde{g})$.

First, the wealth tax is assessed on wealth held at the start of the period, so the liability is $\tau_W W_{i,t}$ and:

$$W_{i,t+1} = W_{i,t}^{\text{pre}} - \tau_W W_{i,t} = W_{i,t}\left(\exp(g_{i,t}) - \tau_W\right) \tag{19}$$

so post-tax log growth is:

$$\tilde{g}^W = \log(\exp(g) - \tau_W) \tag{20}$$

The wealth-tax expression is defined where $\exp(g) > \tau_W$, and the simulation caps each liability at the agent's pre-tax wealth, a cap that never binds at any effective tax rate the paper reports. That expression is concave in g, with derivative $\exp(g)/(\exp(g) - \tau_W)$ exceeding one everywhere, so it lowers the mean and raises the variance:

$$\tilde{\sigma}_W^2 > \sigma^2 \tag{21}$$

The base is what makes the wealth tax widen the growth distribution. A charge fixed in proportion to opening wealth is a larger proportional loss after a bad draw than after a good one, so the tax instrument amplifies the variance of growth. At the baseline calibration the amplification is twelve percent. That base is a stock of wealth at a valuation date, the base Norway, Spain and Switzerland use and the one used here. A tax on end-of-period wealth would give the pure shift $\tilde{g}^W = g + \log(1 - \tau_W)$ and leave variance unchanged.

We ran that convention beside this one. Post-tax variance under an end-of-period wealth tax is 0.090 at every effective tax rate from one to five percent, which is σ^2, while under the valuation-date base it rises from 0.092 to 0.101. Both orderings hold under either convention at all five tax rates. The end-of-period gaps run between 0.73 and 0.80 of the valuation-date ones: at a five percent effective tax rate the wealth inequality gap is 0.066 against 0.085 and the wealth mobility gap -0.014 against -0.019. The base therefore sets how large the gaps are and not which way they point.

Second, for the unrealized capital gains tax, the tax applies only to positive gains. Since gains are proportional to $\exp(g) - 1$, taxation occurs if $g > 0$. Post-tax wealth is:

$$W_{i,t+1} = W_{i,t} \left(\exp(g_{i,t}) - \tau_U \max\{\exp(g_{i,t}) - 1, 0\}\right) \tag{22}$$

meaning that the post-tax gross growth factor equals:

$$\exp(\tilde{g}^U) = \exp(g) - \tau_U \max\{\exp(g) - 1, 0\} = \begin{cases} (1 - \tau_U) \exp(g) + \tau_U & \text{if } g > 0 \\ \exp(g) & \text{if } g \leq 0 \end{cases} \tag{23}$$

For $g > 0$, this simplifies to $e^{\tilde{g}^U} = e^g - \tau_U(e^g - 1) = (1 - \tau_U)e^g + \tau_U$, where the constant term τ_U reflects that the tax applies only to gains $\left(e^g - 1\right)$ while the principal is untaxed. Taking logs, the post-tax log growth rate is:

$$\tilde{g}^U = \begin{cases} \log((1 - \tau_U) \exp(g) + \tau_U) & \text{if } g > 0 \\ g & \text{if } g \leq 0 \end{cases} \tag{24}$$

Positive realizations are thus transformed by a mapping of g with slope below one (in levels, the right tail of $\exp(g)$ is compressed), while negative realizations pass through unchanged. This asymmetric compression reduces variance. To compute $\tilde{\sigma}_U^2$, we use the law of total variance:

$$\mathrm{Var}(\tilde{g}^U) = \mathbb{E}[\mathrm{Var}(\tilde{g}^U \mid S)] + \mathrm{Var}(\mathbb{E}[\tilde{g}^U \mid S]) \tag{25}$$

where $S = \mathbf{1}\{g > 0\}$. Let $p = \Pr(g > 0) = \Phi(\mu/\sigma)$. For the truncated normal $g \sim \mathcal{N}(\mu, \sigma^2)$, define:

$$\mu^+ = \mathbb{E}[g \mid g > 0], \quad \mu^- = \mathbb{E}[g \mid g \leq 0] \tag{26}$$

$$v^+ = \mathrm{Var}(g \mid g > 0), \quad v^- = \mathrm{Var}(g \mid g \leq 0) \tag{27}$$

The post-tax log growth rate has conditional moments of its own on the positive region:

$$\tilde{\mu}_U^+ = \mathbb{E}[\tilde{g}^U \mid g > 0], \qquad \tilde{v}_U^+ = \mathrm{Var}(\tilde{g}^U \mid g > 0) \tag{28}$$

On the negative region, $\tilde{g}^U = g$, so the corresponding moments are $\mathbb{E}[\tilde{g}^U \mid g \leq 0] = \mu^-$ and $\mathrm{Var}(\tilde{g}^U \mid g \leq 0) = v^-$. Applying Equation (25) and comparing to the untaxed case yields:

$$\tilde{\sigma}_U^2 = \tilde{v}_U^+ p + v^-(1-p) + p(1-p)\left[\tilde{\mu}_U^+ - \mu^-\right]^2 \tag{29}$$

$$\sigma^2 = v^+ p + v^-(1-p) + p(1-p)\left[\mu^+ - \mu^-\right]^2 \tag{30}$$

The key difference is that on the positive region $g > 0$, post-tax log wealth growth is:

$$\tilde{g}^U = \log((1-\tau_U)\exp(g) + \tau_U)$$

The capital gains map is a strictly increasing and strictly convex function of g for $\tau_U > 0$, with slope strictly between 0 and 1. The slope and not the curvature does the work below. The mapping is therefore a contraction that compresses variance: $\tilde{v}_U^+ < v^+$. Moreover, the between-state mean gap is reduced, $\tilde{\mu}_U^+ - \mu^- < \mu^+ - \mu^-$. For $\tau_U > 0$ both fall, and the negative-region term is untouched because the mapping is the identity there, so nothing offsets them:

$$\tilde{\sigma}_U^2 < \sigma^2 \tag{31}$$

Combining these results, $\tilde{\sigma}_W^2 > \sigma^2 > \tilde{\sigma}_U^2$, and in particular the variance ordering (confirmed in our simulations):

$$\tilde{\sigma}_W^2 > \tilde{\sigma}_U^2 \tag{32}$$

B.2 Stationary distribution

We show that lower variance implies a higher Pareto tail exponent ζ, and therefore lower wealth inequality. Intuitively, with less dispersed growth shocks, extreme wealth accumulation becomes less likely, producing a thinner right tail.

Kesten process & Pareto tail From the wealth accumulation equation (Equation (3)), wealth evolves as:

$$W_{i,t+1} = A_{i,t} W_{i,t} + B_{i,t} \tag{33}$$

where $A_{i,t} = \exp(\tilde{g}_{i,t})$ is the multiplicative component and $B_{i,t} = \mathcal{T}_t^{\text{redist}}$ is the additive component. Wealth therefore follows a Kesten process. Redistribution ensures mean reversion: wealthy agents pay more in taxes than they receive in transfers. Under regularity conditions, Kesten (1973) established that the stationary distribution has a Pareto right tail:

$$\Pr(W > w) \sim C \cdot w^{-\zeta} \quad \text{as } w \to \infty \tag{34}$$

The tail exponent $\zeta > 0$ solves $\mathbb{E}[A_t^{\zeta}] = 1$, so it is set by the multiplicative component alone: a wider spread of A_t lowers ζ and a larger drag raises it. Redistribution enters through B_t, which is what makes the stationary law exist rather than what fixes its exponent.

Variance and tail exponent Holding redistribution fixed, lower dispersion of A_t increases ζ. The condition $\mathbb{E}[A_t^{\zeta}] = 1$ pins down ζ. If A_t has less dispersion, its ζ-th moment is smaller for any given ζ, so a higher ζ is needed to satisfy the condition. In our setting, dispersion differences across tax instruments are driven by the variance of post-tax log growth $\tilde{\sigma}^2 = \text{Var}(\tilde{g})$, which fixes the dispersion of $A = \exp(\tilde{g})$ one for one. At equal ETR, variance differs but aggregate redistribution intensity is held constant. Concretely, write mean reversion as $\mathbb{E}[\tilde{g}] = -\gamma$ with $\gamma > 0$. Because $A = \exp(\tilde{g})$, the tail condition is a moment generating function evaluated at ζ. If post-tax log growth were Gaussian, $\tilde{g} \sim \mathcal{N}(-\gamma, \tilde{\sigma}^2)$, then $\mathbb{E}[A^{\zeta}] = \exp(-\zeta\gamma + \frac{1}{2}\zeta^2\tilde{\sigma}^2)$ and $\mathbb{E}[A^{\zeta}] = 1$ has the single non-zero root:

$$\zeta = \frac{2\gamma}{\tilde{\sigma}^2} \tag{35}$$

Mean reversion in logs is not mean reversion in levels. By Jensen's inequality the two differ by the variance drag that multiplicative shocks impose on the geometric mean. Writing

$$\lambda \equiv -\log \mathbb{E}[A] = -\log(\exp(r - \rho) - \text{ETR}) \tag{36}$$

for the deterministic drag, which is what the tax takes net of what the portfolio earns, the decomposition is exact under the same Gaussian approximation:

$$\gamma = \lambda + \tfrac{1}{2}\tilde{\sigma}^2 \tag{37}$$

The deterministic drag is the same under both tax instruments, so everything that separates them sits in $\tilde{\sigma}^2$. Each tax instrument takes ETR $\cdot\, W_{i,t}$ in expectation from a base of $W_{i,t}$, so $\mathbb{E}[A] = \mathbb{E}[\exp(g)] - \text{ETR}$ whichever is in force. The drift includes the Jensen correction, so $\mathbb{E}[\exp(g)] = \exp(r - \rho)$ for any σ. Equation (36) then follows from the definition of the effective tax rate in Equation (8). That definition leaves λ depending only on $r - \rho$ and the effective tax rate: identical at a common ETR, and invariant to σ. Substituting Equation (37) into Equation (35):

$$\zeta = 1 + \frac{2\lambda}{\tilde{\sigma}^2} \tag{38}$$

Equation (38) says that ζ falls as $\tilde{\sigma}^2$ rises while the tax take is held fixed, so the variance ordering of Appendix B.1 determines the wealth inequality ordering on its own. That reading carries the same domain as the mobility condition below: it needs the drag λ to be positive, and so an effective tax rate above $\exp(r - \rho) - 1 \approx 2.0$ percent. Below that tax rate λ turns negative and the exponents order the other way, 0.78 against 0.76 at one percent, while the simulations put the wealth tax ahead at every tax rate we run. The closed form and the simulation part company there, and the paper reports the simulation. Neither tax instrument leaves post-tax growth exactly Gaussian, so the equation is an approximation. Solving $\mathbb{E}[A^{\zeta}] = 1$ numerically at the baseline calibration puts it within one percent of the true root under the wealth tax and within six percent under the capital gains tax.

Variance compression is the primary driver of the wealth inequality differences the simulations report. The deterministic drag λ is common to both tax instruments at a matched effective tax rate, and γ differs only through $\tilde{\sigma}^2$. Where $\lambda > 0$, the variance ordering $\tilde{\sigma}^2_W > \tilde{\sigma}^2_U$ therefore implies:

$$\zeta_W < \zeta_U \tag{39}$$

B.3 Wealth mobility

We derive the probability that an agent exits the top of the distribution, and the condition under which the tax instrument that leaves more variance is also the more mobile one.

The comparison is between an agent and a quantile threshold, not between two agents. Wealth mobility is the probability of exiting the top wealth decile over a horizon h, and the threshold is a feature of a stationary distribution that stays put. The agent's log wealth drifts and diffuses relative to it.

Let $d = \log W_{i,t} - \log w_{90,t}$ be the log distance from an agent in the top wealth decile down to the threshold $w_{90,t}$. Under the Pareto tail of Appendix B.2, d is exponentially distributed with rate ζ. For an agent in the top wealth decile the per-capita transfer is small relative to its wealth. Its log wealth therefore evolves as $\log W_{i,t+h} - \log W_{i,t} = \sum_{s=0}^{h-1} \tilde{g}_{i,t+s}$, which by the central limit theorem is approximately Gaussian:

$$\log W_{i,t+h} - \log W_{i,t} \sim \mathcal{N}\left(-\gamma h,\ h\tilde{\sigma}^2\right) \tag{40}$$

The agent has exited the decile at horizon h when this change falls short of $-d$, so, integrating over the position of the agent within the decile:

$$P(\text{exit at } h) = \mathbb{E}_d\left[\Phi\left(\frac{\gamma h - d}{\tilde{\sigma}\sqrt{h}}\right)\right], \qquad d \sim \text{Exp}(\zeta) \tag{41}$$

Three quantities set the exit probability. Drift moves the agent toward the threshold at rate γ, diffusion of scale $\tilde{\sigma}\sqrt{h}$ moves it either way, and ζ sets how far it has to travel. The expectation in Equation (41) is a Gaussian–exponential convolution and has a closed form. Writing $a = \gamma\sqrt{h}/\tilde{\sigma}$:

$$P(\text{exit at } h) = \Phi(a) - \exp\left(\tfrac{1}{2}\zeta^2 h\tilde{\sigma}^2 - \zeta\gamma h\right)\Phi\left(a - \zeta\tilde{\sigma}\sqrt{h}\right) \tag{42}$$

The tail exponent collapses this expression. Substituting Equation (35) sets $\zeta\tilde{\sigma}\sqrt{h} = 2a$ and $\frac{1}{2}\zeta^2 h\tilde{\sigma}^2 = \zeta\gamma h = 2a^2$, so the exponential factor is one and the second term becomes $\Phi(-a)$. Equation (42) collapses to:

$$P(\text{exit at } h) = 2\,\Phi\left(\frac{\gamma\sqrt{h}}{\tilde{\sigma}}\right) - 1 \tag{43}$$

Wealth mobility therefore depends on the tax instrument through the single ratio $\gamma/\tilde{\sigma}$, and Equation (43) is increasing in it. Substituting the decomposition of Equation (37) shows what that ratio contains:

$$P(\text{exit at } h) = 2\,\Phi\left(\sqrt{h}\left[\frac{\lambda}{\tilde{\sigma}} + \frac{\tilde{\sigma}}{2}\right]\right) - 1 \tag{44}$$

Volatility moves wealth mobility two ways, and above $\tilde{\sigma} = \sqrt{2\lambda}$ the second way wins. The first term of the bracket falls in $\tilde{\sigma}$: more post-tax volatility means a thicker tail, and a thicker tail puts the threshold further away. The second rises in $\tilde{\sigma}$: more of it means larger movements toward it. Their sum is minimized at $\tilde{\sigma} = \sqrt{2\lambda}$ for $\lambda > 0$, so wealth mobility is lowest there and rises in $\tilde{\sigma}$ above it. At equal ETR λ is common, so the comparison is a comparison of $\tilde{\sigma}$

alone. The tax instrument that leaves more variance is then the more mobile one, provided both sit on the rising branch, for which it is enough that the more compressing tax instrument does:

$$\tilde{\sigma}_U^2 > 2\lambda \tag{45}$$

The condition asks that the compressing tax instrument still leave enough post-tax volatility to sit above the deterministic drag. Equation (45) therefore states the trade-off as a condition, not as the outcome of one parameterization. Through Equation (38) it is equivalent to $\zeta_U < 2$, the infinite-variance Pareto case. That equivalence uses the approximate exponent: at our baseline the exact root is 2.03 against the approximation's 1.93, so the two sit on opposite sides of the threshold. Read the condition on $\tilde{\sigma}^2$, where both sides are measured in the same way. When it holds and the increments are independent of d, the ordering follows:

$$P(\text{exit})_W > P(\text{exit})_U \tag{46}$$

Read as a statement about the tail, the condition is not obviously demanding. Surveying estimates for the United States, Benhabib & Bisin (2018) report tail indices between 1.48 and 1.55 once differential nonresponse among the very rich is corrected for, comfortably below two. The comparison is a plausibility check, not a test. ζ_U is this model's exponent under a capital gains tax, and the measured figure is the exponent of an actual distribution under whatever taxes it faces.

The derivation assumes an agent's growth does not depend on how far it sits from the threshold, and one extension in Section 4 removes that. Equation (41) treats the increments $\tilde{g}_{i,t+s}$ as independent of d, so every agent in the decile faces the same growth distribution whatever its history. A tax instrument whose liability depends on an agent's own past breaks it, and a capital gains tax with a loss offset is that tax instrument. The exit probability then cannot be read off γ and $\tilde{\sigma}$ alone, however comfortably the condition is satisfied.

Range of validity The closed forms track the simulation closely. Post-tax growth is not exactly Gaussian and the top wealth decile is not exactly Pareto at a finite horizon, so Equations (38) and (43) are not identities. At a common effective tax rate of 5% the closed form puts the probability of staying at 0.256 under the wealth tax and 0.271 under the capital gains tax at the baseline. Simulation gives 0.263 and 0.285 (Table 1), a predicted gap of 1.5 points against 2.2.

They disagree only about the crossing. In simulation the ordering reverses between $\sigma = 0.175$ and $\sigma = 0.20$. At the lower combination the wealth mobility gap is half a standard error from zero and at the upper one it is four and a half. The simulations place it no more finely than one step in σ. Equation (45) places the boundary at $\sigma \approx 0.29$, and Section 3.4 traces it across effective tax rates as well.

The condition is sufficient and loose: it understates the range over which the trade-off holds. The calibrated $\sigma = 0.30$ satisfies it under the Gaussian form, though not with much room. Under the exact root it sits just outside (2.03 against a threshold of 2), where the trade-off still holds in simulation. The simulated trade-off, which is what Section 3 reports, holds well below the calibration.

The condition also has a domain. It needs the drag λ to be positive, and so an effective tax rate above $\exp(r - \rho) - 1 \approx 2.0$ percent, below which the tail exponent falls under one and mean wealth is infinite. It weakens as the effective tax rate falls towards that edge: 2λ falls to zero, so the condition is satisfied at every volatility, while the simulated boundary does not fall with it. Nothing between two and three percent was simulated, so where the two diverge is not located.

The simulated boundary runs from about 0.10 at a one percent effective tax rate to about 0.20 at five percent. Bach et al. (2020) measure 6% at the ninth decile, 8.7% for the top 2.5–1 percent and 27.5% for the top 0.01 percent. Only the last clears the boundary at every effective tax rate we simulate.

Those brackets are diversified portfolios, and this model is about undiversified ones. The same paper puts private equity at 51% for the median holder and about 40% at the top, so the calibration clears the boundary with room. The trade-off is a statement about concentrated wealth, which is where these taxes bind and where a Pareto exponent and a top wealth decile exit rate mean anything. That restriction is a scope condition and we state it as one.

Volatility falling through the distribution The runs above hold volatility constant across the distribution, and Bach et al. measure it rising toward the top. A further run lets σ rise linearly from 0.06 in the bottom decile to 0.30 in the top. It puts no step at the top wealth decile, keeps the scale-dependent return gradient of Appendix C in place, and calibrates both tax instruments to five effective tax rates over 25 simulations.

The wealth mobility ordering does not resolve at the bottom of the effective tax rate range, and it reverses above it. At one percent the wealth inequality gap is 2.8 points and the wealth mobility gap 0.7 points at 1.6 standard errors, short of the two the simulations above treat as resolved. At two percent the gap is 0.1 points at 0.4 standard errors, which does not resolve. At three, four and five percent the top wealth decile is less mobile under the wealth tax than under the capital gains tax, by 1.4, 4.4 and 7.3 points (8, 42 and 66 standard errors). The wealth inequality gap keeps widening to 17.3 points.

With volatility falling through the distribution the trade-off holds, unresolved, only at the one percent Norway charges, which is the scope condition above in a sharper form. Ten deciles over ten thousand agents cannot express the gradient inside the top wealth decile, where most of the measured variation sits, so the run bounds the condition rather than locating it.

C Type and scale dependence

The baseline assumes identical expected return r and volatility σ for all agents. We consider two extensions. First, with type dependence, agents are assigned to one of K types with expected return r_k and volatility σ_k. Agent i's wealth evolves as:

$$W_{i,t+1} = W_{i,t} \exp(\mu_{k_{i,t}} + \sigma_{k_{i,t}} \varepsilon_{i,t}) - \mathcal{T}_{i,t} + \mathcal{T}_t^{\text{redist}} \tag{47}$$

where $\mu_k = r_k - \rho - \frac{1}{2}\sigma_k^2$ is the drift implied by expected return r_k. Types evolve according to a symmetric Markov chain with persistence π: agents remain in their current type with probability π and transition uniformly otherwise. Second, with scale dependence, returns depend on current wealth through the decile. Let $q_{i,t} \in \{1, \ldots, 10\}$ denote agent i's decile. Wealth evolves as:

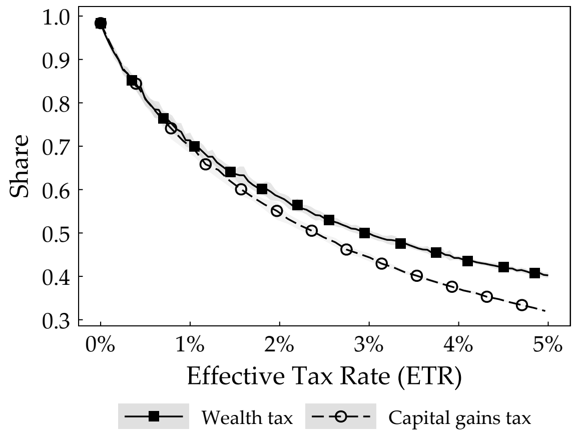
$$W_{i,t+1} = W_{i,t} \exp(\mu(q_{i,t}) + \sigma(q_{i,t})\varepsilon_{i,t}) - \mathcal{T}_{i,t} + \mathcal{T}_t^{\text{redist}} \tag{48}$$

where $\mu(q) = r(q) - \rho - \frac{1}{2}\sigma(q)^2$ and $r(q)$ is increasing in q: under scale dependence, richer agents earn higher expected returns.

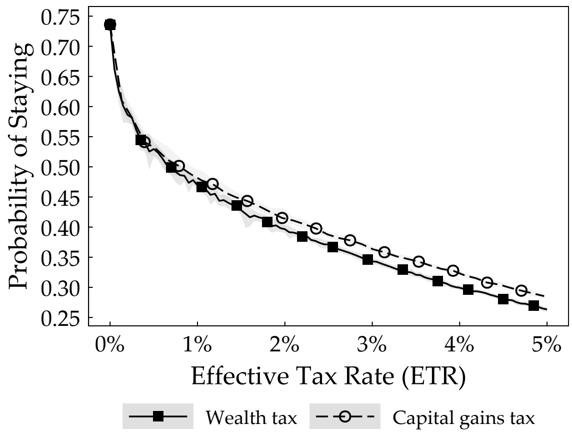
The trade-off holds under return heterogeneity (Figure 7). Under type dependence and under scale dependence alike, the capital gains tax generates lower wealth inequality and lower wealth mobility than the wealth tax. Both orderings resolve at every combination: the weakest is scale dependence at one percent, where the wealth inequality gap is 2.6 standard errors from zero and the wealth mobility gap 3.2. The tax rates compared are matched effective tax rates from one to five percent.

Both extensions increase wealth inequality and reduce wealth mobility relative to the baseline (Figure 8). The fall in wealth mobility follows from the thicker tail. By Appendix B.3 the log distance from an agent in the top wealth decile down to the threshold is exponential with rate ζ. A lower exponent puts that threshold further away, and fewer agents cross it.

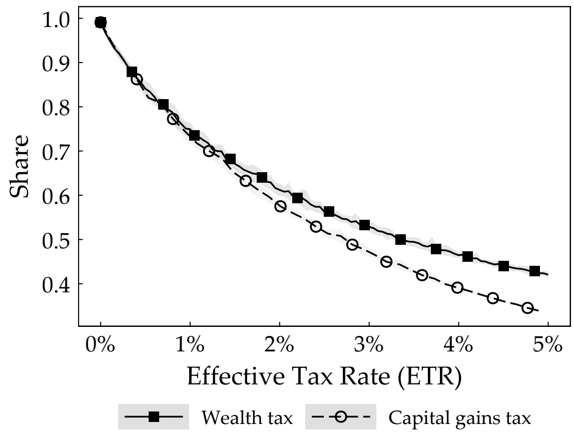
Figure 7: The two tax instruments compared under type and scale dependence in returns.



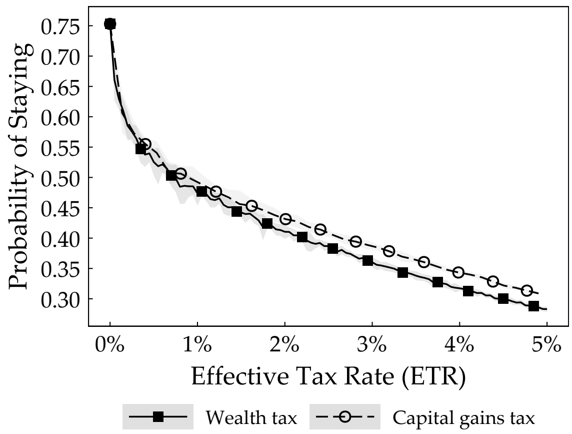
(a) Wealth inequality, baseline



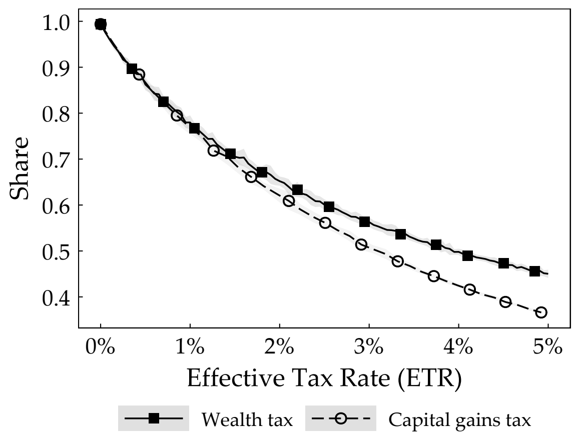
(b) Wealth mobility, baseline



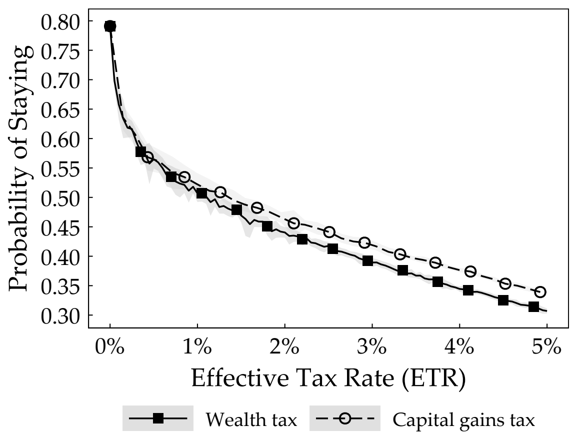
(c) Wealth inequality, type-dependent



(d) Wealth mobility, type-dependent



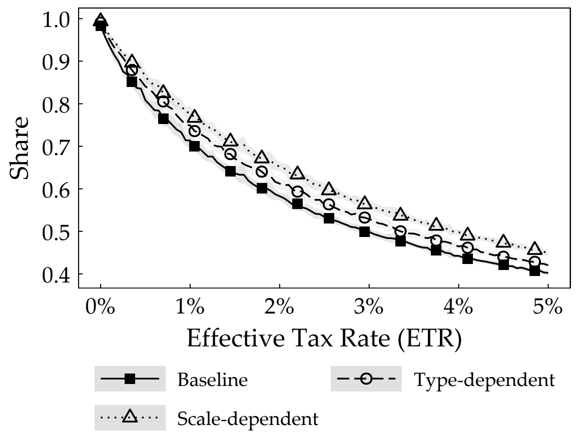
(e) Wealth inequality, scale-dependent



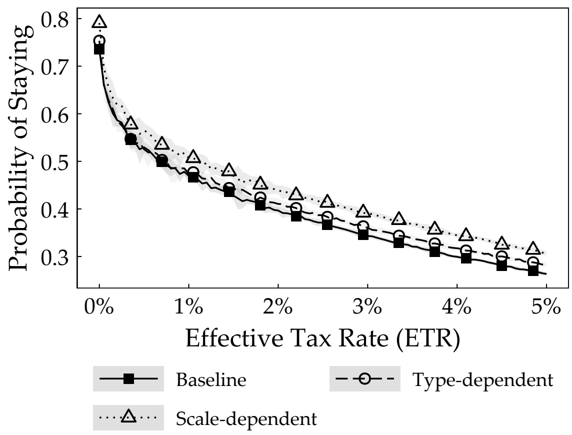
(f) Wealth mobility, scale-dependent

Notes: Each row shows a different scenario. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Type dependence: $K = 5$ types with $r_k \in \{0.06, 0.07, 0.08, 0.09, 0.10\}$, $\sigma = 0.30$, persistence $\pi = 0.99$. Scale dependence: $r(q)$ increasing linearly from 0.06 to 0.10 across deciles, $\sigma = 0.30$. Parameters: $N = 10{,}000$, $T = 200$, $\rho = 0.06$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation (barely visible due to low cross-run variance).

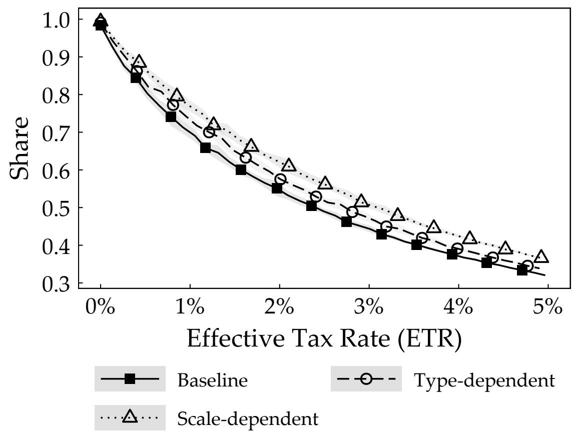
Figure 8: Effect of type- and scale-dependent returns on wealth inequality and wealth mobility by tax instrument.



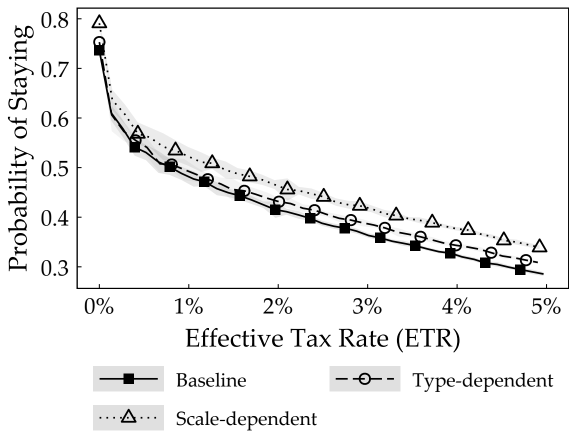
(a) Wealth inequality, wealth tax



(b) Wealth mobility, wealth tax



(c) Wealth inequality, unrealized CGT



(d) Wealth mobility, unrealized CGT

Notes: Each row shows a different tax instrument. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Lines compare baseline (homogeneous returns), type-dependent, and scale-dependent returns. Both extensions increase wealth inequality and reduce wealth mobility. Type dependence: $K = 5$ types with $r_k \in \{0.06, 0.07, 0.08, 0.09, 0.10\}$ at persistence $\pi = 0.99$. Scale dependence: $r(q)$ increasing linearly from 0.06 to 0.10 across deciles. Parameters: $\sigma = 0.30$, $\rho = 0.06$, $N = 10{,}000$, $T = 200$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the baseline, dashed lines with open circles type-dependent returns, and dotted lines with open triangles scale-dependent returns. Shaded areas show ± 1 standard deviation (barely visible due to low cross-run variance).

D Hand-to-mouth agents

A fraction $\psi \in (0, 1]$ of agents are entrepreneurs who operate the investment technology. The remaining fraction $1 - \psi$ are hand-to-mouth (HtM) agents with zero wealth. Agent types are fixed at $t = 0$. Entrepreneur i's wealth evolves as in the baseline:

$$W_{i,t+1} = W_{i,t} \exp(\mu + \sigma \varepsilon_{i,t}) - \mathcal{T}_{i,t} + \mathcal{T}_t^{\text{redist}} \tag{49}$$

HtM agents never accumulate: they hold $W_{i,t}^{\text{HtM}} = 0$ for all t, receiving transfers and consuming them immediately. Tax revenue is collected only from entrepreneurs but redistributed equally across all N agents:

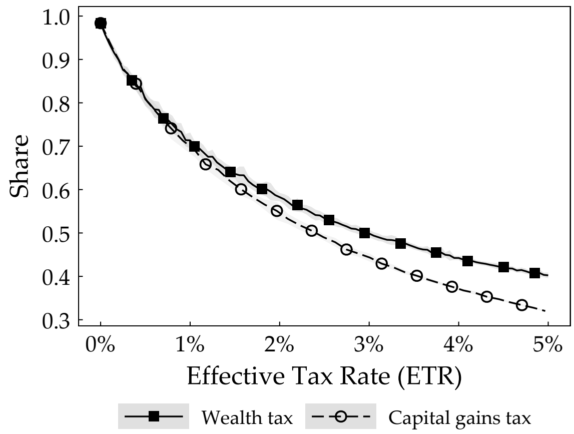
$$\mathcal{T}_t^{\text{redist}} = \frac{1}{N} \sum_{j \in \text{entrepreneurs}} \mathcal{T}_{j,t} \tag{50}$$

The runs below set $\psi \in \{1.0, 0.9, 0.7\}$, where $\psi = 1$ is the baseline with entrepreneurs-only.

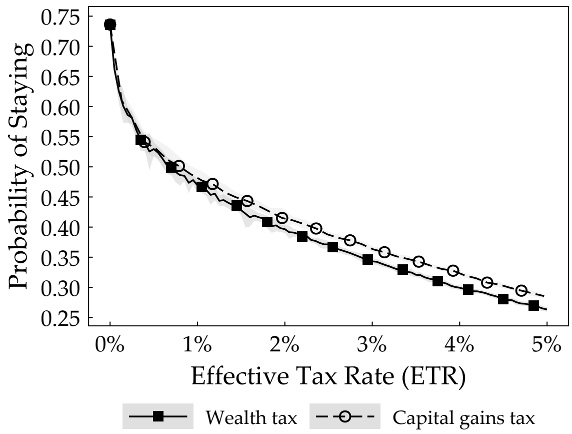
The trade-off holds when some agents are hand-to-mouth (Figure 9). At every entrepreneur share the capital gains tax generates lower wealth inequality and lower wealth mobility than the wealth tax. Both orderings hold at all fifteen combinations and both resolve against sampling error, which is what separates this extension from the two that follow it. Measured at matched effective tax rates, the weakest combination is $\psi = 0.9$ at a one percent effective tax rate. There the wealth inequality gap is 2.5 standard errors from zero and the wealth mobility gap 3.3.

Higher hand-to-mouth shares, meaning lower ψ, increase wealth inequality and reduce wealth mobility (Figure 10).

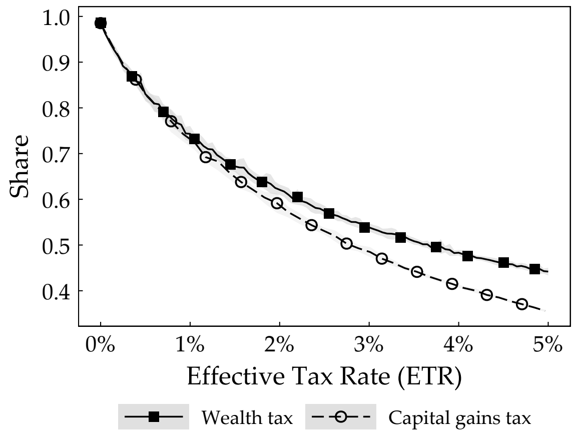
Figure 9: The two tax instruments compared at three entrepreneur shares.



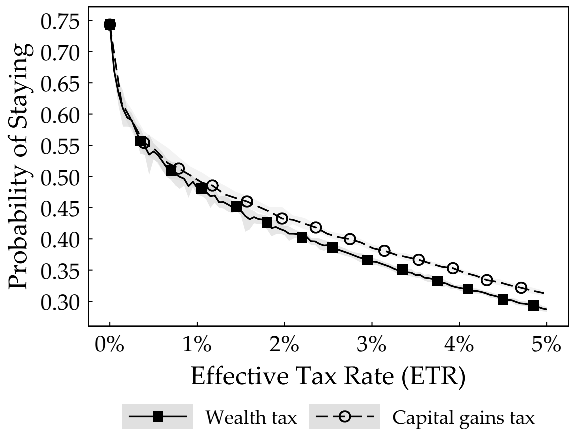
(a) Wealth inequality, $\psi = 1.0$



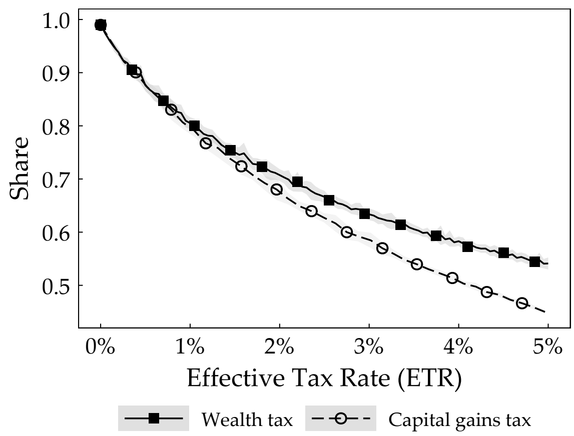
(b) Wealth mobility, $\psi = 1.0$



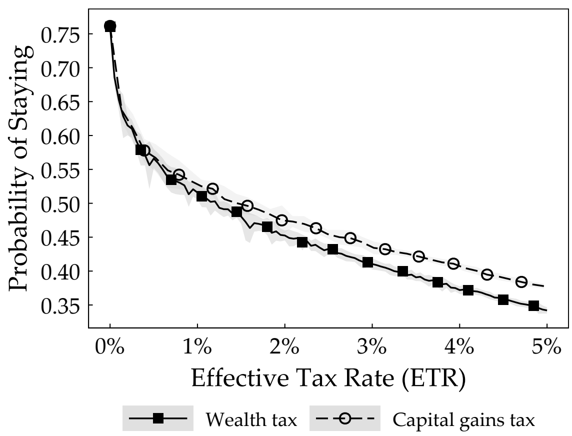
(c) Wealth inequality, $\psi = 0.9$



(d) Wealth mobility, $\psi = 0.9$



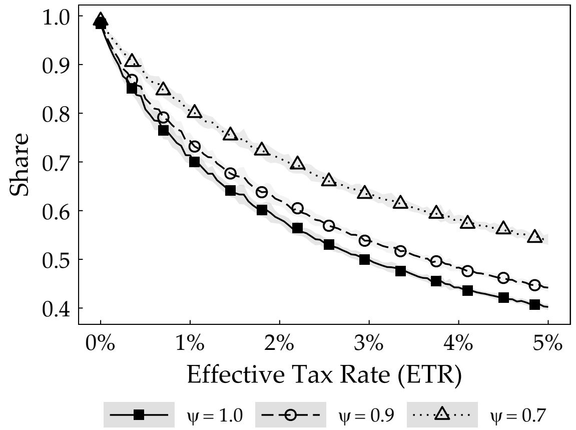
(e) Wealth inequality, $\psi = 0.7$



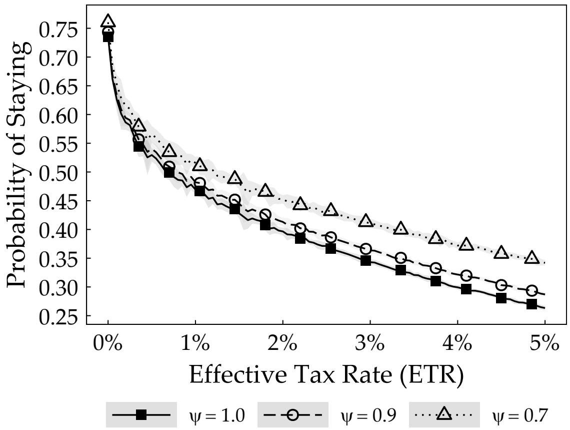
(f) Wealth mobility, $\psi = 0.7$

Notes: Each row shows a different entrepreneur share ψ. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Outcomes are computed over the full population, and HtM agents are always ranked at the bottom. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation (barely visible due to low cross-run variance).

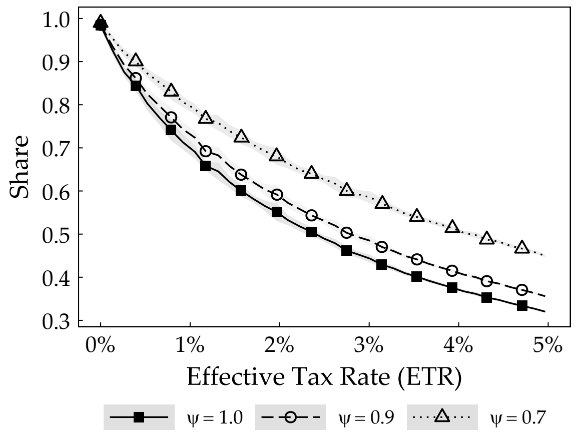
Figure 10: Effect of the entrepreneur share on wealth inequality and wealth mobility by tax instrument.



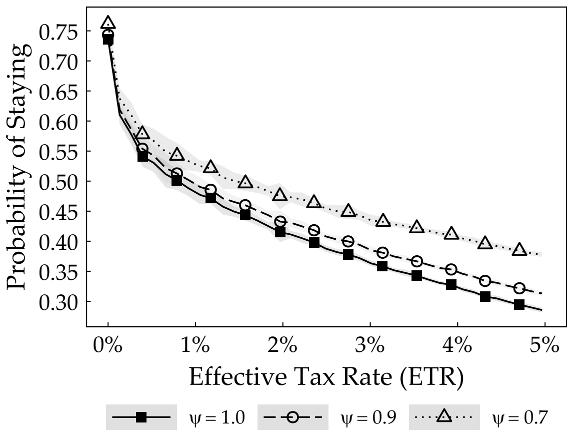
(a) Wealth inequality, wealth tax



(b) Wealth mobility, wealth tax



(c) Wealth inequality, unrealized CGT



(d) Wealth mobility, unrealized CGT

Notes: Each row shows a different tax instrument. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Lines compare $\psi \in \{1.0, 0.9, 0.7\}$. Lower ψ (more HtM agents) increases wealth inequality and reduces wealth mobility, because tax revenue collected from entrepreneurs is shared across all agents and less of it returns to those who paid it. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show $\psi = 1.0$, dashed lines with open circles $\psi = 0.9$, and dotted lines with open triangles $\psi = 0.7$. Shaded areas show ± 1 standard deviation (barely visible due to low cross-run variance).

E Aggregate investment risk

We extend the baseline model to include aggregate shocks affecting all agents. Agent i's wealth evolves as:

$$W_{i,t+1} = W_{i,t} \exp(\mu + \sigma_{\text{idio}} \varepsilon_{i,t} + \sigma_{\text{agg}} \eta_t) - \mathcal{T}_{i,t} + \mathcal{T}_t^{\text{redist}} \tag{51}$$

where $\varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0,1)$ is idiosyncratic and $\eta_t \overset{iid}{\sim} \mathcal{N}(0,1)$ is common to all agents. The total variance is preserved:

$$\sigma_{\text{idio}} = \sqrt{\kappa}\,\sigma, \qquad \sigma_{\text{agg}} = \sqrt{1-\kappa}\,\sigma \tag{52}$$

The split holds total variance fixed, $\sigma_{\text{idio}}^2 + \sigma_{\text{agg}}^2 = \sigma^2$, and the parameter $\kappa \in [0,1]$ governs the idiosyncratic share: $\kappa = 1$ recovers the baseline, while $\kappa = 0$ implies pure aggregate risk where all agents move together. The runs below set $\kappa \in \{1.0, 0.9, 0.8\}$.

The trade-off holds under aggregate risk (Figure 11). The wealth mobility ordering holds at every idiosyncratic share and every effective tax rate. Under the capital gains tax agents are less likely to exit the top wealth decile than under the wealth tax. The wealth inequality ordering holds everywhere the simulations resolve it, and fails to resolve at two of the fifteen combinations, both at a one percent effective tax rate and both under aggregate risk. There the gap stands at $+0.0015$ for $\kappa = 0.9$ and -0.0069 for $\kappa = 0.8$, which are 0.3 and 1.5 standard errors from zero, so unresolved and not reversed.

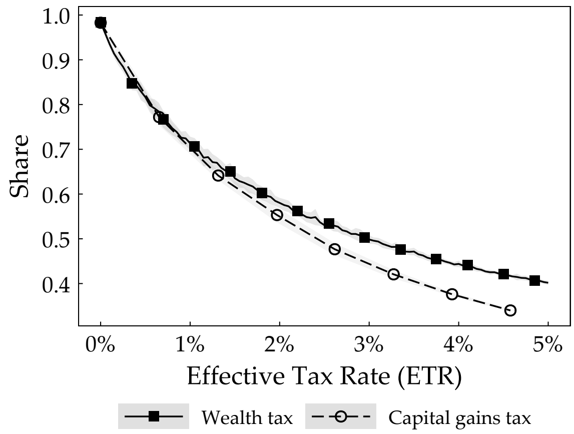
At $\kappa = 1$ the same effective tax rate gives a full percentage point at 2.4 standard errors, which is the weakest resolved gap in standard errors anywhere we simulate. Smaller gaps in points resolve elsewhere, at a third of a point in the volatility sweep. On this appendix's coarser set of capital gains statutory tax rates that is a smaller gap than the 2.0 points the baseline run's matched combination gives Section 3.2. The capital gains schedule steps from an effective tax rate of 0.7% to 1.3% in that corner, across a stretch in which the top wealth decile's share falls by thirteen points. The comparison there is interpolated and not measured. Section 3.4 reports the same feature of the baseline simulations, where gaps near a boundary are small against sampling error and should be read as a band.

The two gaps move in opposite directions as the idiosyncratic share falls. At every effective tax rate compared the wealth inequality gap narrows, from 8.1 to 5.8 percentage points at a five percent effective tax rate in this appendix's own simulations. The wealth mobility gap widens over the same fall, from 2.0 to 3.0 points at that effective tax rate. Table 1's baseline row, at matched combinations, gives 8.2 and 2.2, and its refined $\kappa = 0.8$ row the same 5.8 and

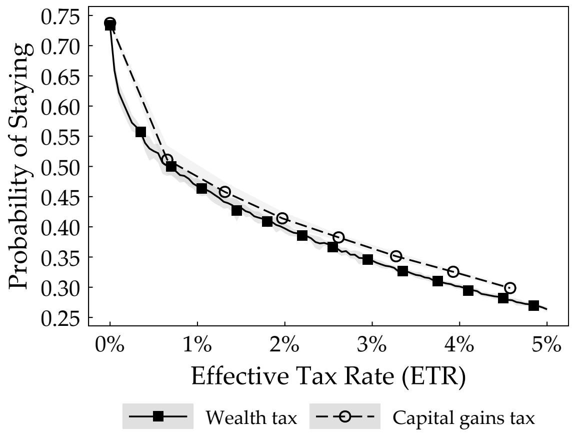
3.0 to a tenth of a point. The trade-off therefore holds across idiosyncratic shares wherever the simulations resolve it.

Lower κ reduces both wealth inequality and wealth mobility, because agents move together (Figure 12).

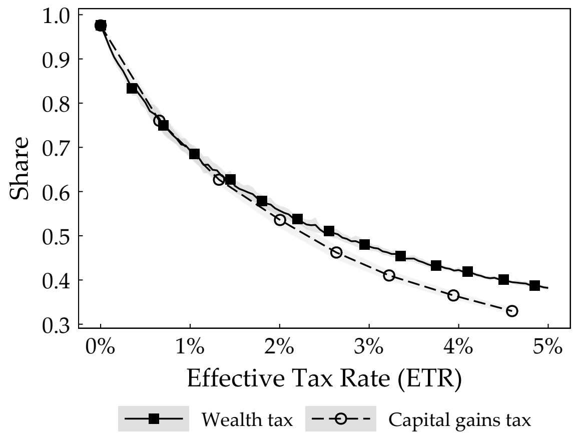
Figure 11: The two tax instruments compared at three idiosyncratic shares.



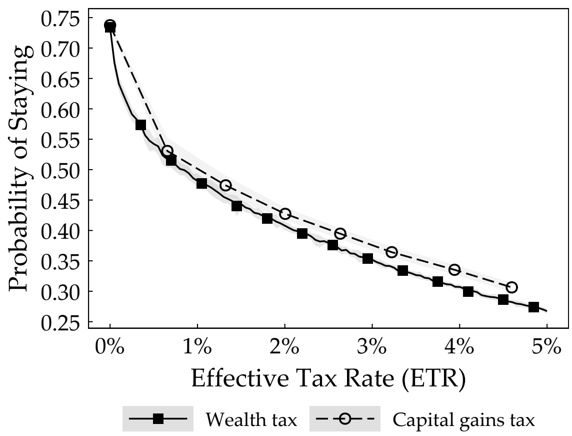
(a) Wealth inequality, $\kappa = 1.0$



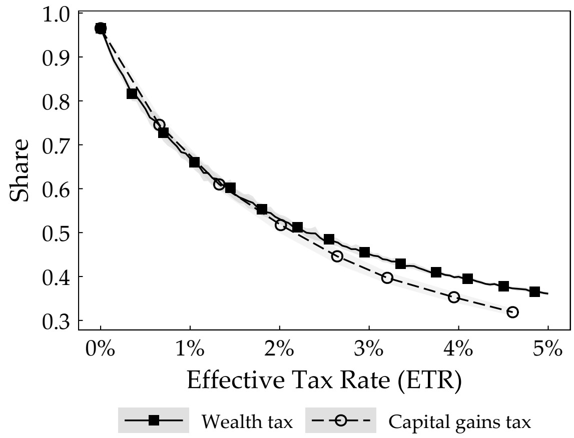
(b) Wealth mobility, $\kappa = 1.0$



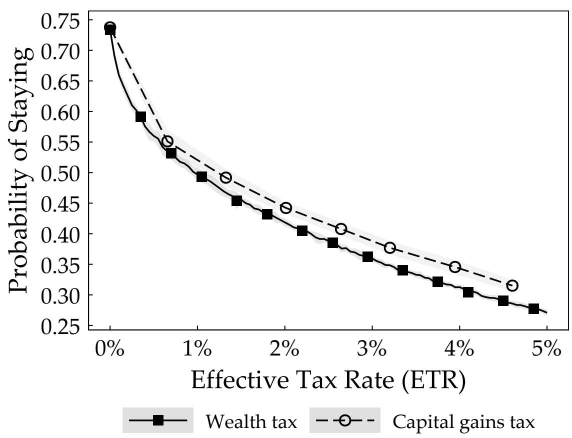
(c) Wealth inequality, $\kappa = 0.9$



(d) Wealth mobility, $\kappa = 0.9$



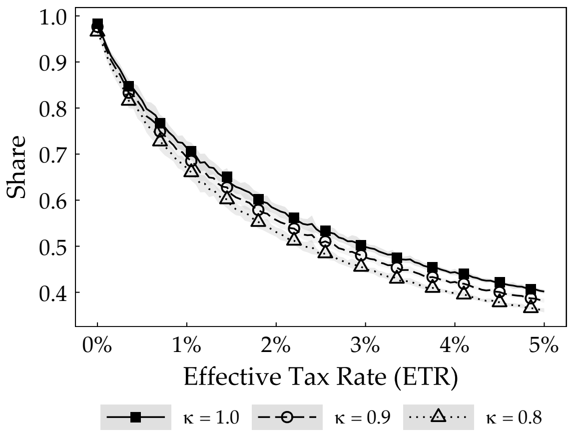
(e) Wealth inequality, $\kappa = 0.8$



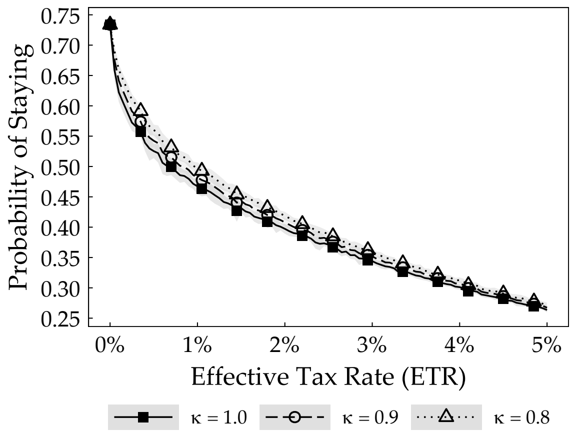
(f) Wealth mobility, $\kappa = 0.8$

Notes: Each row shows a different idiosyncratic share κ. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation (often barely visible due to low cross-run variance).

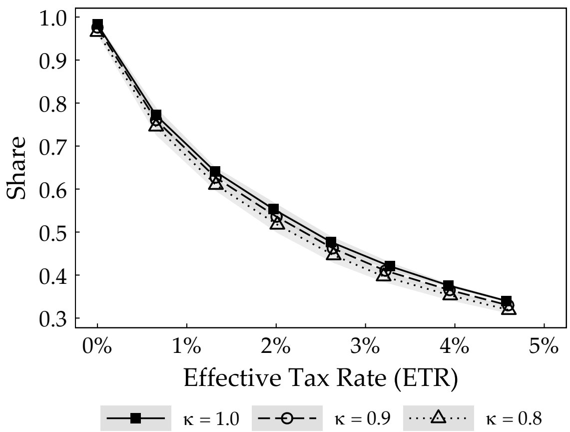
Figure 12: Effect of idiosyncratic share on wealth inequality and wealth mobility by tax instrument.



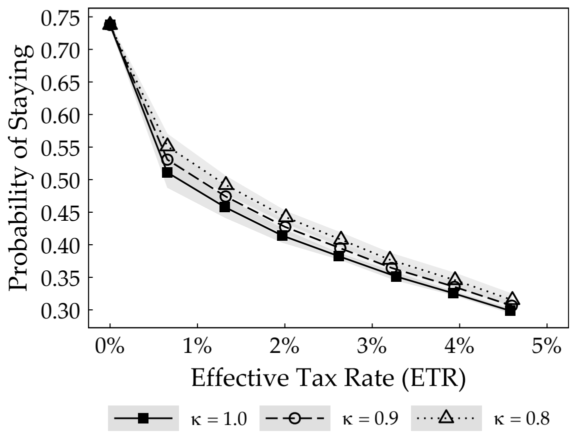
(a) Wealth inequality, wealth tax



(b) Wealth mobility, wealth tax



(c) Wealth inequality, unrealized CGT



(d) Wealth mobility, unrealized CGT

Notes: Each row shows a different tax instrument. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Lines compare $\kappa \in \{1.0, 0.9, 0.8\}$. Lower κ (more aggregate risk) reduces both wealth inequality and wealth mobility. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show $\kappa = 1.0$, dashed lines with open circles $\kappa = 0.9$, and dotted lines with open triangles $\kappa = 0.8$. Shaded areas show ± 1 standard deviation (often barely visible due to low cross-run variance).

F Tax progressivity

We introduce tax progressivity through tax exemption thresholds. For the wealth tax, only wealth above $\chi \cdot \bar{W}_t$ is taxed:

$$\mathcal{T}_{i,t}^{(W)} = \tau_W \max\{W_{i,t} - \chi \cdot \bar{W}_t, 0\} \tag{53}$$

For the unrealized capital gains tax, only capital gains above χ times mean positive gains are taxed:

$$\mathcal{T}_{i,t}^{(U)} = \tau_U \max\{\Delta W_{i,t}^{\text{pre}} - \chi \cdot \overline{\Delta W^+}_t, 0\} \tag{54}$$

where $\Delta W_{i,t}^{\text{pre}} = W_{i,t}^{\text{pre}} - W_{i,t}$ and $\overline{\Delta W^+}_t = \mathbb{E}[\Delta W_{i,t}^{\text{pre}} \mid \Delta W_{i,t}^{\text{pre}} > 0]$.

The parameter $\chi \geq 0$ determines tax progressivity: $\chi = 0$ recovers the baseline, while $\chi > 0$ exempts agents below the threshold. Let $\phi \in (0, 1]$ denote the fraction of agents above the tax threshold. Higher χ implies lower ϕ and requires higher statutory tax rates to achieve the same effective tax rate ETR. The runs below set $\chi \in \{0, 0.5, 1\}$.

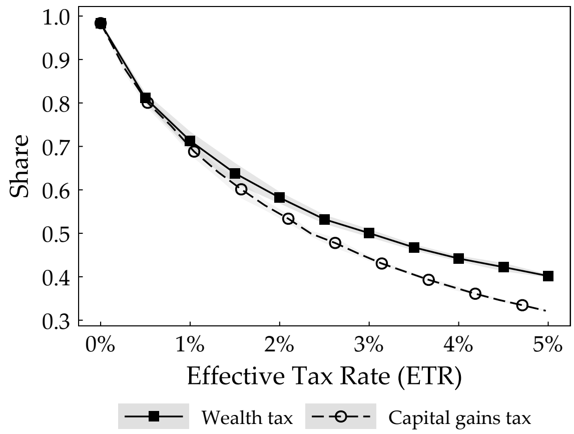
The inequality–mobility trade-off from Section 3 persists because the variance ordering holds within the taxed population. Under the progressive wealth tax, the agents above the threshold face mean reversion without variance compression. Under the progressive capital gains tax, they face both mean reversion and variance compression. The ordering $\tilde{\sigma}_W^2 > \tilde{\sigma}_U^2$ holds at any tax threshold level.

The trade-off holds under an exemption threshold (Figure 13). The wealth mobility ordering holds at every threshold and every effective tax rate, and holds comfortably. The capital gains tax leaves agents less likely to exit the top wealth decile at all fifteen combinations, and the smallest of those gaps is 4.5 standard errors from zero. The wealth inequality ordering holds everywhere the simulations resolve it, and fails to resolve at two of the fifteen, both at a one percent effective tax rate and both under an exemption. There the gap stands at $+0.0030$ for $\chi = 0.5$ and $+0.0007$ for $\chi = 1$, which are 0.6 and 0.1 standard errors from zero.

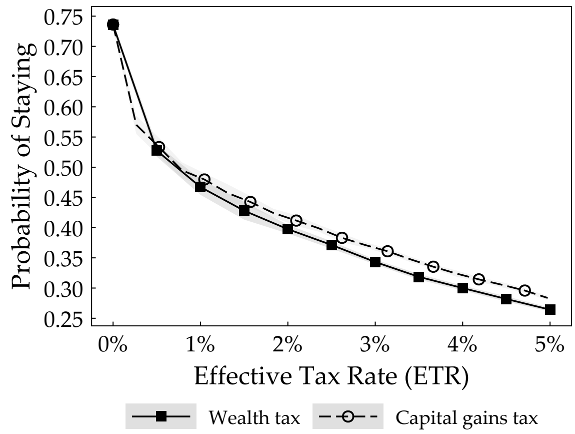
The sign is right at both and the simulations do not resolve it. A one percent effective tax rate on a small taxed population should give exactly that: neither tax instrument moves the distribution far enough there to tell the two apart from the variation between runs. Appendix E reports the same feature at the same effective tax rate.

Higher exemption thresholds reduce wealth inequality and need higher statutory tax rates to reach the same effective tax rate (Figure 14).

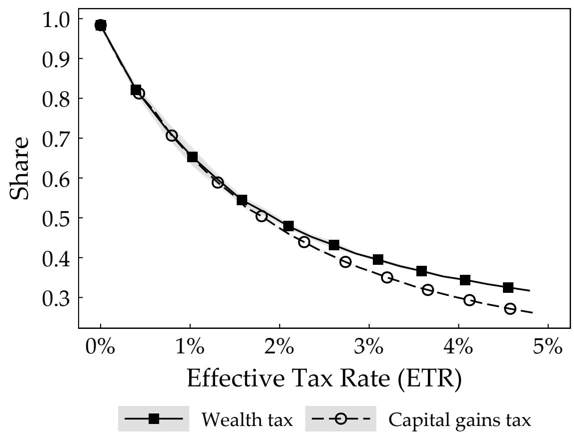
Figure 13: The two tax instruments compared at three exemption thresholds.



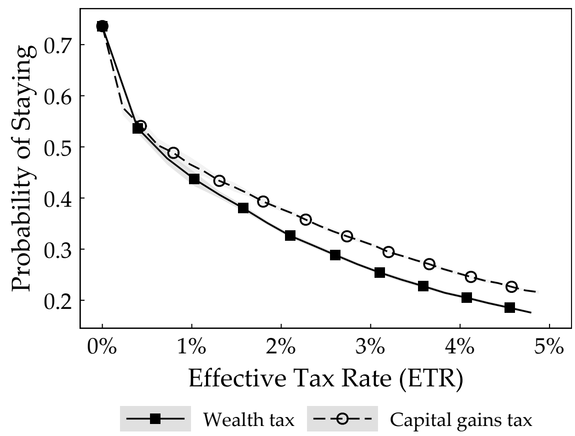
(a) Wealth inequality, $\chi = 0$



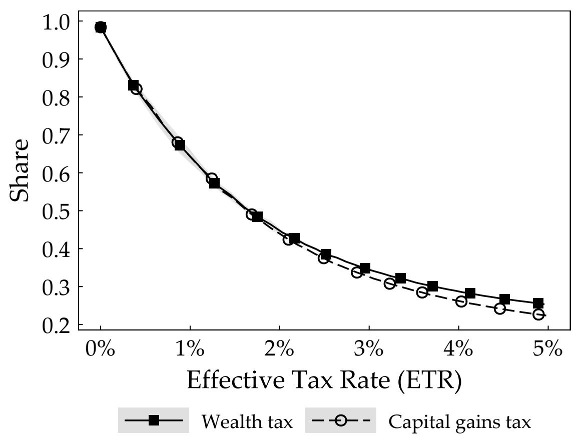
(b) Wealth mobility, $\chi = 0$



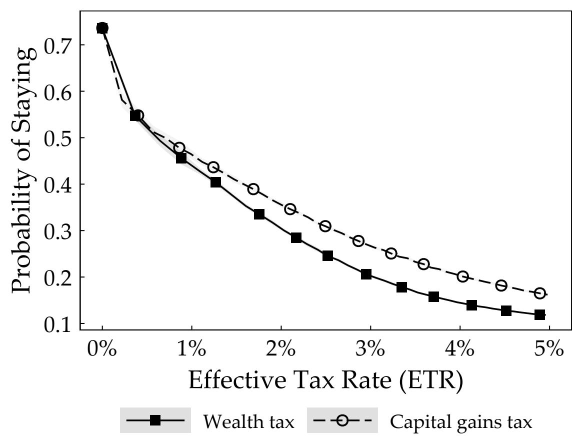
(c) Wealth inequality, $\chi = 0.5$



(d) Wealth mobility, $\chi = 0.5$



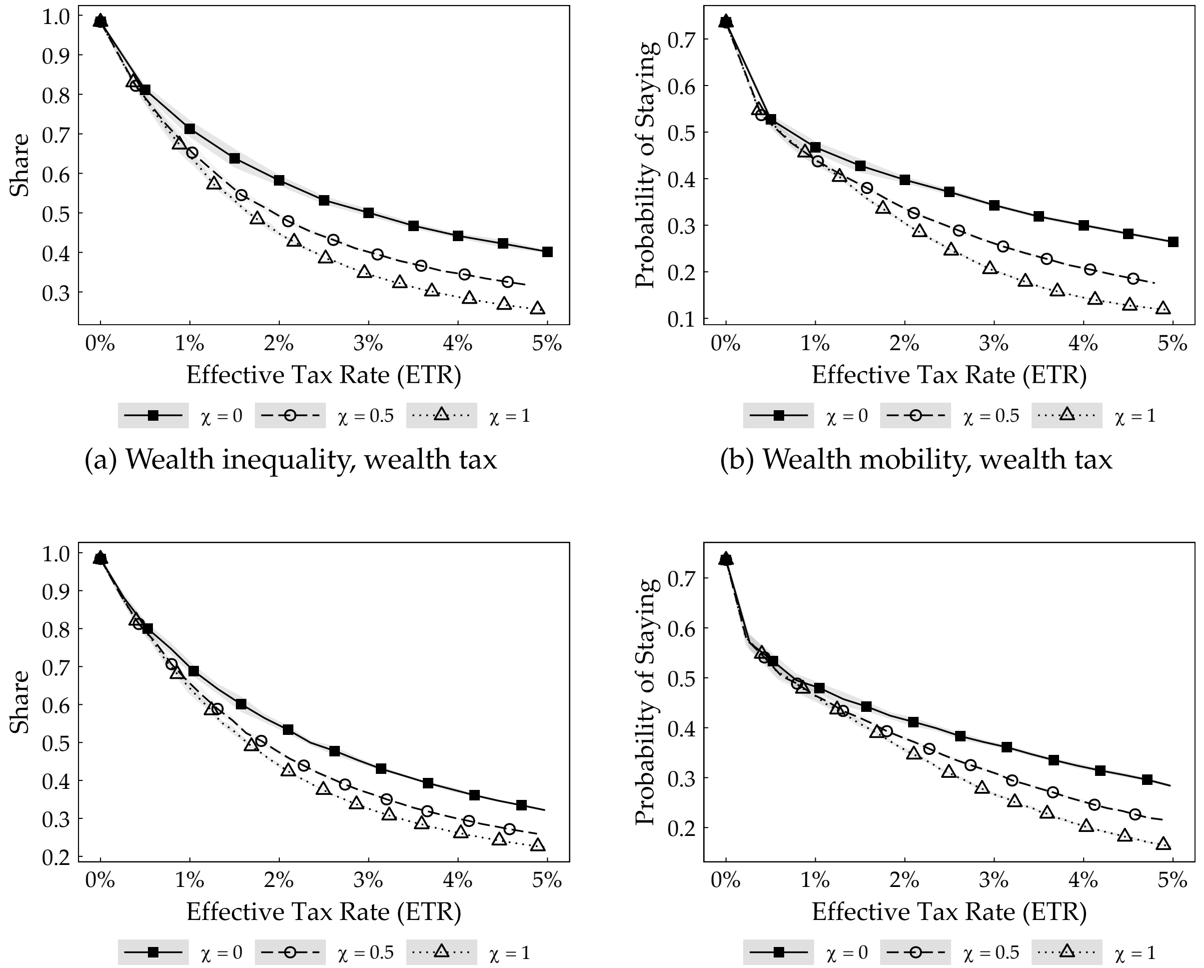
(e) Wealth inequality, $\chi = 1$



(f) Wealth mobility, $\chi = 1$

Notes: Each row shows a different threshold level χ. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show the wealth tax, and dashed lines with open circles the unrealized capital gains tax. Shaded areas show ± 1 standard deviation (often barely visible due to low cross-run variance).

Figure 14: Effect of progressivity on wealth inequality and wealth mobility by tax instrument.



(a) Wealth inequality, wealth tax

(b) Wealth mobility, wealth tax

(c) Wealth inequality, unrealized CGT

(d) Wealth mobility, unrealized CGT

Notes: Each row shows a different tax instrument. Left panels: top 10% wealth share. Right panels: probability of staying in top 10% after 20 periods. Lines compare $\chi \in \{0, 0.5, 1\}$. Higher χ reduces wealth inequality and requires higher statutory tax rates to achieve the same ETR. Parameters: $N = 10{,}000$, $T = 200$, $r = 0.08$, $\rho = 0.06$, $\sigma = 0.30$. Each data point is the average over 25 independent simulations at a given statutory tax rate, plotted at the effective tax rate those simulations achieve. Solid lines with filled squares show $\chi = 0$, dashed lines with open circles $\chi = 0.5$, and dotted lines with open triangles $\chi = 1$. Shaded areas show ± 1 standard deviation (often barely visible due to low cross-run variance).

G Elasticity of intertemporal substitution

The elasticity moves the drift of wealth and leaves its variance alone. With CRRA utility $u(c) = c^{1-\theta}/(1-\theta)$, i.i.d. gross returns R and no labor income, wealth evolves as $W' = (W-c)R$. Guess $V(W) = JW^{1-\theta}/(1-\theta)$. The first-order condition of $\max_c\{c^{1-\theta}/(1-\theta) + \beta J\,\mathbb{E}[R^{1-\theta}](W-c)^{1-\theta}/(1-\theta)\}$ is $c^{-\theta} = \beta J\,\mathbb{E}[R^{1-\theta}](W-c)^{-\theta}$, so c is proportional to W: write $c = \omega W$. Substituting the guess back and solving for J gives $\beta\,\mathbb{E}[R^{1-\theta}] = (1-\omega)^{\theta}$, hence

$$\omega = 1 - \left[\beta\,\mathbb{E}(R^{1-\theta})\right]^{1/\theta} \tag{55}$$

Because ω does not depend on W, wealth remains multiplicative, $W' = (1-\omega)WR$, so the elasticity of intertemporal substitution $1/\theta$ shifts the drift by $\log(1-\omega)$ and leaves $\mathrm{Var}(\log R) = \sigma^2$ untouched at every value, with $\omega = 1-\beta$ at $\theta = 1$. The value function is finite exactly when $\beta\,\mathbb{E}(R^{1-\theta}) < 1$, which is the condition that bounds the exercise below.

We give both tax instruments the same consumption rate, which keeps the drag common to them (Section 3.3; Table 4, behind the elasticity paragraph of Section 4). We compute ω on pre-tax returns and pass it to the simulation as an effective rate of time preference $-\log(1-\omega)$, which reproduces the drift the rule implies. The capital gains statutory tax rate is recalibrated at each θ, since the drift moves the base of that tax. At $\theta = 1$ the rule returns $\omega = 0.0566$ against the baseline's $\rho = 0.06$, a difference in drift of under two thousandths, which is why the $\theta = 1$ row reproduces the baseline. An agent re-optimising under each tax instrument's post-tax return would choose an ω that differs between them.

We solved that case as well. Post-tax wealth stays multiplicative under either tax instrument, so a constant consumption rate still solves the problem and the rate can be found separately for each. At $\theta = 1$ the two rules already separate, to 0.054 under the wealth tax and 0.066 under the capital gains tax against the common rule's 0.0566, because each tax instrument enters the growth factor and the pre-tax log rule no longer solves the problem. At an elasticity of 2 they separate widely, to 0.098 under the wealth tax and 0.175 under the capital gains tax, and the statutory capital gains tax rate matching a five percent effective tax rate rises from 0.38 to 0.74. Both orderings survive: the wealth inequality gap widens to 0.115 from the 0.082 of the common rule and the wealth mobility gap narrows to -0.016 from -0.020, at 87.9 and 10.8 standard errors. Letting the agent respond to the tax instrument therefore moves the magnitudes and leaves the signs.

Table 4: The elasticity of intertemporal substitution (EIS), under the common consumption rule.

θ	EIS	ω	τ_U	$\tilde{\sigma}^2_W$	$\tilde{\sigma}^2_U$	Top 10%, W / U	Stay, W / U
0.50	2.00	0.0573	0.381	0.1006	0.0652	0.403 / 0.321	0.266 / 0.286
1.00	1.00	0.0566	0.379	0.1006	0.0653	0.403 / 0.321	0.266 / 0.287
2.00	0.50	0.0238	0.326	0.1002	0.0668	0.409 / 0.332	0.272 / 0.292
2.50	0.40	0.0038	0.299	0.0999	0.0677	0.412 / 0.339	0.275 / 0.295

Notes: W is the wealth tax and U the unrealized capital gains tax. Both orderings resolve at every θ: the smallest of the eight gaps is 10.1 standard errors from zero, taken over the same ten runs with the two tax instruments' variances added. Variances are computed by quadrature on the post-tax growth maps. The wealth shares and the probabilities of staying in the top wealth decile after 20 periods are averages over 10 independent simulations. Pre-tax variance is $\sigma^2 = 0.09$ at every row, which is the point: the EIS moves the drift and not the variance. Parameters otherwise as in the baseline, at a five percent effective tax rate. The runs stop at $\theta = 2.5$ because $\beta\,\mathbb{E}(R^{1-\theta})$, on pre-tax returns, reaches one at $\theta = 2.6$, beyond which the infinite-horizon problem has no interior solution at $\sigma = 0.30$.

It also shortens the range. The interior solution disappears at $\theta = 2.59$ on pre-tax returns, which is the boundary this table stops short of, but at $\theta = 1.92$ under a five percent wealth tax and at $\theta = 1.76$ under the capital gains tax rate that matches it. The two rows below an elasticity of 0.6 therefore have no re-optimised counterpart, and taxing the return is what removes them.

The variance ordering is untouched at every elasticity. Post-tax variance under the wealth tax stays at 0.100 and under the capital gains tax between 0.065 and 0.068 across elasticities from 2 down to 0.4, against $\sigma^2 = 0.09$ throughout. The wealth inequality gap narrows as the elasticity falls, from 0.08 to 0.07, while the wealth mobility gap holds at 0.02. A lower elasticity means less consumption and so a higher drift, which lowers λ. Below an elasticity of about one half λ is negative. The closed form of Section 3.3 is silent there, so the narrowing is the simulation's.

The limit on this exercise comes from the setting, not from the tax comparison. The value function converges only if $\beta\,\mathbb{E}(R^{1-\theta}) < 1$, and with θ above one the term $\frac{1}{2}(1-\theta)^2\sigma^2$ grows quadratically, so at high volatility the condition fails at moderate risk aversion. At $\sigma = 0.30$ the boundary is $\theta = 2.6$, an EIS of 0.4, and at $\sigma = 0.15$ it falls to an EIS of 0.1. Below the boundary an agent with one risky asset, no labor income and an infinite horizon has an unbounded precautionary motive and no interior solution.

H Redistribution rule

The redistribution rule moves the level of wealth inequality and not the ordering between tax instruments. We return a fraction α of tax revenue as an equal transfer and the remainder in proportion to post-tax wealth (Table 5, behind the redistribution-rule paragraph of Section 4).

The reason is where each part of the transfer sits in the growth process. An equal transfer is additive and sits in B of the Kesten form $W' = AW + B$, where the tail exponent does not see it. A transfer proportional to wealth returns part of the tax in proportion and belongs in A, so each agent pays α of the effective tax rate in multiplicative terms. The simulation matches that prediction to within 10^{-4} for the wealth tax and within 3×10^{-4} for the capital gains tax at every α tested. The ordering is untouched because λ is common to both tax instruments.

The wealth mobility gap widens throughout, from 2.1 to 6.1 percentage points as α falls from one to a quarter, while the wealth inequality gap rises to $\alpha = 0.5$ and edges back at a quarter. Equation (16) anticipates the first direction inside its domain: a lower λ lowers the boundary the compressing tax instrument has to clear. At $\alpha = 0.25$ the drag is negative, the closed form is silent, and the gap there is the simulation's.

The table separates the two parts. The proportional part is multiplicative and belongs inside the growth factor, so the column headed λ leaves it there and the next column is $-\log(\exp(r - \rho) - \alpha\,\mathrm{ETR})$.

Table 5: The redistribution rule.

α	Tax instrument	ETR	λ	Predicted	Top 10%	Stay
1.00	Wealth tax	0.050	0.0303	0.0303	0.404	0.264
	Capital gains tax	0.050	0.0300	0.0303	0.322	0.284
0.75	Wealth tax	0.050	0.0175	0.0175	0.463	0.307
	Capital gains tax	0.050	0.0173	0.0175	0.370	0.337
0.50	Wealth tax	0.050	0.0049	0.0048	0.550	0.362
	Capital gains tax	0.050	0.0047	0.0048	0.448	0.406
0.25	Wealth tax	0.050	−0.0077	−0.0077	0.690	0.438
	Capital gains tax	0.050	−0.0077	−0.0077	0.592	0.499

Notes: Averages over 10 independent simulations, in a run separate from Table 1's. "Stay" is the probability of staying in the top wealth decile after 20 periods, so the wealth tax being the more mobile tax instrument shows as the lower number. λ is the drag measured from the simulated panel and "Predicted" is $-\log(\exp(r-\rho)-\alpha\,\mathrm{ETR})$. The gaps quoted in the text are computed before rounding. Both orderings resolve at every α: the smallest of the eight gaps is 7.3 standard errors from zero, taken over the same ten runs with the two tax instruments' variances added. A negative λ means the expected growth factor net of tax exceeds one. The distribution is still stationary, because stationarity requires $\gamma = \lambda + \tilde{\sigma}^2/2 > 0$ rather than $\lambda > 0$, but by Equation (38) the tail exponent has fallen below one and the distribution no longer has a finite mean. Parameters otherwise as in the baseline, at a 5% effective tax rate.

I Autocorrelated shocks

Both orderings hold at every autocorrelation tested, and the tail thickens as persistence rises (Table 6, behind the autocorrelated-shocks paragraph of Section 4).

The capital gains statutory tax rate is recalibrated at each ρ_ε, because persistence widens the base of that tax. Persistence makes the next draw of an agent with good recent draws likely good too, so wealth and realized capital gains become positively correlated and the wealth-weighted base widens. The statutory tax rate needed for a 5% effective tax rate falls from 38% to 17%. Carrying the baseline statutory tax rate over instead would have set a 5% wealth tax against a 9% capital gains tax at $\rho_\varepsilon = 0.75$, which raises different tax revenue and would have flattered the ordering reported here.

The exponents should be read as movement across ρ_ε, not as levels. They are Hill estimates on a finite cross-section after two hundred periods, and at $\rho_\varepsilon = 0$ the estimate of 1.83 sits above the exact Kesten root of 1.61 for the same parameters, so the levels carry an upward finite-sample bias. By $\rho_\varepsilon = 0.75$ persistence has taken the exponent below one, which is a distribution with no finite mean.

The closed forms of Appendix B rest on Kesten's theorem, which is stated for independent multipliers, and we do not claim they extend. Results for Markov-modulated multiplicative processes exist, and Beare & Toda (2022) determine the Pareto exponent for them. Those results are for processes stabilized by reset rather than by redistribution, which is not this model. What Section 4 reports for autocorrelated shocks is therefore a simulation result and not an analytical one.

Table 6: Autocorrelated shocks, both tax instruments at a 5% effective tax rate.

ρ_ε	Tax instrument	Statutory tax rate	ETR	Top 10%	Stay	Hill (5%)	Hill (1%)
0.00	Wealth tax	0.050	0.050	0.401	0.263	1.83	1.90
	Capital gains tax	0.383	0.050	0.320	0.283	2.39	2.53
0.50	Wealth tax	0.050	0.050	0.643	0.215	1.19	1.22
	Capital gains tax	0.282	0.049	0.550	0.254	1.37	1.39
0.75	Wealth tax	0.050	0.050	0.815	0.181	0.94	0.93
	Capital gains tax	0.169	0.048	0.780	0.213	0.99	0.96

Notes: Averages over 10 independent simulations, in a run separate from Table 1's. "Stay" is the probability of staying in the top wealth decile after 20 periods, so the wealth tax being the more mobile tax instrument shows as the lower number. Hill estimates are taken on the final-period cross-section at the stated tail fraction. Both orderings resolve at every row: the smallest of the six gaps is 9.1 standard errors from zero, taken over the same ten runs with the two tax instruments' variances added. Parameters otherwise as in the baseline. At $\rho_\varepsilon = 0.75$ the capital gains tax lands at 4.8% rather than 5%, because the effective tax rate is estimated with more noise once the tail is that heavy. The shortfall works against the ordering reported, since a less heavily taxed capital gains tax is the less compressed one.

J Capital loss offsets

An unlimited offset for capital losses reverses the wealth mobility ordering above a two percent effective tax rate (Table 7, behind the loss-offset paragraph of Section 4).

A loss is set against later capital gains without limit or expiry, and against nothing else. Each agent carries a balance $L_{i,t}$ of unrelieved losses, so the base is $\max\{(W_{i,t}^{\text{pre}} - W_{i,t}) - L_{i,t}, 0\}$ and the balance rises by each period's loss and falls by each period's gain, floored at zero. A loss is not set against other income in the period it falls.

The offset changes the base, so the statutory tax rate cannot be carried over. The wealth tax's statutory tax rate equals its effective tax rate by construction, and the capital gains statutory tax rate is solved for each effective tax rate by root-finding on the measured effective tax rate. The base shrinks as the statutory tax rate rises, so no statutory tax rate reaches an effective tax rate above 4.1%, which is why the table stops at four.

The reversal needs statutory tax rates well above those the no-offset comparison uses. Under a full offset a 2% effective tax rate needs a statutory tax rate of 44% on net capital gains and 4% needs 96%, against the 38% that a five percent effective tax rate takes in Section 2.

The gap effect beats the variance effect here. The gap effect wins elsewhere too, below the growth volatility threshold of Section 3.4, where the mobility condition fails. Only here does it win with both the variance ordering and that condition intact. An offset shields whoever has lost and taxes whoever has only gained, so it falls hardest on agents climbing through the tail. The top wealth decile's share under the capital gains tax drops from 0.44 without an offset to 0.29 with it at a 3% effective tax rate. A thinner tail leaves the top wealth decile less spread out, so an agent in it sits closer to the threshold. That closeness is the gap effect of Section 3, and here it beats the variance effect, because the liability now depends on where an agent sits.

What fails is neither the variance ordering nor the wealth mobility condition. Post-tax variance is 0.10 under the wealth tax against 0.08 with the offset, so the variance ordering survives. And at a 3% effective tax rate λ is 0.0098, so the condition asks for a capital gains variance above 0.020 and it is 0.084. Both hold, and the ordering reverses anyway. What the offset breaks is an assumption behind Equation (14), which treats an agent's log wealth increments as independent of where it sits in the distribution. An agent deep in the tail has had a run of gains and no balance to shelter behind, so it pays the full statutory tax rate. One near the threshold has recent losses and pays nothing. The drift becomes position-dependent, and the

Table 7: Capital loss offsets across the effective tax rate.

	No offset			Unlimited offset		
ETR	τ_U	Ineq. gap	Stay gap	τ_U	Ineq. gap	Stay gap
1%	0.077	+0.018	−0.013	0.210	+0.071	−0.013
2%	0.154	+0.041	−0.019	0.443	+0.157	+0.003
3%	0.230	+0.060	−0.022	0.695	+0.212	+0.058
4%	0.307	+0.073	−0.022	0.957	+0.231	+0.160

Notes: Averages over 10 independent simulations. The no-offset rows are a separate run of ten simulations rather than a slice of the simulations behind Section 3.2, so they differ from it by a few thousandths. The wealth inequality gap is the top wealth decile's share under the wealth tax minus that under the capital gains tax. The stay gap is the probability of remaining in the top wealth decile under the wealth tax minus that under the capital gains tax, and it is reported that way round rather than as an exit gap because these runs were built on the stay probability the model records, so it carries the sign of Table 3 and is the negative of the exit-rate gap of Figure 6, and the paper's claim here is a positive wealth inequality gap together with a negative stay gap. Under unlimited loss offset that holds at a 1% effective tax rate, does not resolve at 2%, and fails above. Gaps are measured in standard errors the same way as in the other extension appendices, over the same ten runs with the two tax instruments' variances added: the stay gap runs −4.0, +1.1, +28.2 and +95.5 across the four tax rates, so the 2% cell is the only one of the sixteen in the table inside two. Because the two arms are drawn on common random numbers, that yardstick is conservative. The statutory tax rates show why the upper rows are of limited practical interest: a 4% effective tax rate requires taxing 96% of net gains. Parameters otherwise as in the baseline.

exit probability can no longer be read off γ and $\tilde{\sigma}$ alone. Equation (16) is sufficient given that independence, and this extension removes it.

The extension measures the mechanical effect and not the portfolio response. Adding a loss offset is the move from Domar & Musgrave's (1944) case without loss-sharing to the case with it, and their result is that it encourages risk-taking. That works through portfolio choice, which this model does not have.