Local Gini Estimation under Social Sampling: Rank-Dependent Underestimation and Inequality Order Reversals

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Abstract

We study inequality measurement under socially constrained observability and introduce a local Gini coefficient computed on ego-centered comparison sets, represented as fixed-width rank windows along Pen's income parade. For a linear parade, the discrete sliding-window local Gini admits an exact closed form and is strictly below the population Gini, with rank monotonicity. For curved parades, the paper uses a first-order quantile approximation to derive explicit rank profiles for exponential and Pareto regimes and validates these profiles against finite-sample sliding-window Ginis in simulations. The framework implies systematic rank heterogeneity in locally perceived inequality, inequality order reversals across societies relative to population Ginis (including majority misranking in the Pareto–linear comparison for $1 < \alpha < 2$ in the continuous-rank sense), and subunit-elastic local responses to changes in inequality within the Pareto family.

1 Introduction

How is inequality measured when individuals observe only a socially local subset of incomes rather than a representative sample of the population? This paper develops a formal answer by introducing a local Gini coefficient computed on ego-centered comparison sets. Social sampling is represented in reduced form as a fixed-width rank window along Pen's income parade (Pen, 1971). The resulting object is a tractable statistic of local inequality under homophilic observability.

This perspective is motivated by a large literature documenting systematic distortions in beliefs about inequality and social position. A prominent regularity is middle-class bias in self-placement: respondents across objective positions tend to place themselves near the center of the social hierarchy (Evans and Kelley, 2004). Another is systematic underestimation of inequality, with substantial heterogeneity across income positions. The framework in this paper is a parsimonious measurement-level explanatory attempt for these stylized facts: if individuals observe socially local and non-representative comparison sets, then inequality computed on those sets can be systematically lower than population inequality even without imposing biased information processing. In this sense, the model identifies a parsimonious measurement-level mechanism that may help explain these stylized facts if agents form beliefs about aggregate inequality from their socially local samples and extrapolate from them. This mechanism is consistent with network-based explanations showing that such patterns can arise under local and homophilic information (Schulz et al., 2022), with social structure shaping information and reference groups (Schokkaert and Truyts, 2017).

The contribution of this paper is not a behavioral model of inference, but an analytical characterization of the inequality statistic generated by locally constrained observability. The paper derives rank-dependent local Ginis for a linear benchmark (exactly) and for curved parades (through an explicitly stated first-order quantile approximation), and shows how the resulting profiles depend on parade shape. The framework yields three main results: (i) strict local underestimation relative to the corresponding population Gini for the benchmark objects studied, which provides a parsimonious measurement-level mechanism that may help explain the empirical stylized fact of underestimation if agents extrapolate from socially local samples, (ii) systematic rank heterogeneity including distinct profile shapes across linear, exponential, and Pareto regimes, and (iii) failure of local inequality to preserve cross-society inequality rankings induced by population Ginis, including a majority-misranking region in the Pareto–linear comparison.

The analysis is organized around a clear distinction between exact and approximate results. In the linear parade, the discrete sliding-window local Gini admits an exact closed form. For curved parades, the paper uses a first-order quantile approximation yielding explicit rank profiles which numerical simulations show to provide an excellent fit. Theorems are stated for these approximation formulas, and numerical simulations show that the finite-sample discrete sliding-window Gini exhibits the same qualitative behavior for the parameterizations considered.

2 Model and results

Pen's parade orders individuals from poorest to richest and plots income against rank (Pen, 1971). Let a population of size n have incomes ordered nondecreasingly:

$$y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n)}. \tag{1}$$

An individual is indexed by income rank $r \in \{1, \ldots, n\}$. Social sampling is represented by a window half-width $w \in \mathbb{N}$. The symmetric window is

$$\mathcal{W}^{\mathrm{sym}}(r) = \{r - w, \ldots, r + w\}, \tag{2}$$

with window size

$$N = 2w + 1. \tag{3}$$

Directional variants shift the window while keeping N fixed:

$$\mathcal{W}^{\mathrm{up}}(r) = \{r, \ldots, r + 2w\}, \tag{4}$$

$$\mathcal{W}^{\mathrm{down}}(r) = \{r - 2w, \ldots, r\}. \tag{5}$$

All results below are stated for interior ranks where all indices remain in $\{1, \ldots, n\}$.

Per assumption, rank windows imply a mechanical middle-self perception under symmetric sampling in line with empirical findings. Let $s(r) \in \{1, \ldots, N\}$ denote the within-window rank of the focal individual and define perceived quantile position as

$$\hat{p}(r) = \frac{s(r) - 1}{N - 1}. \tag{6}$$

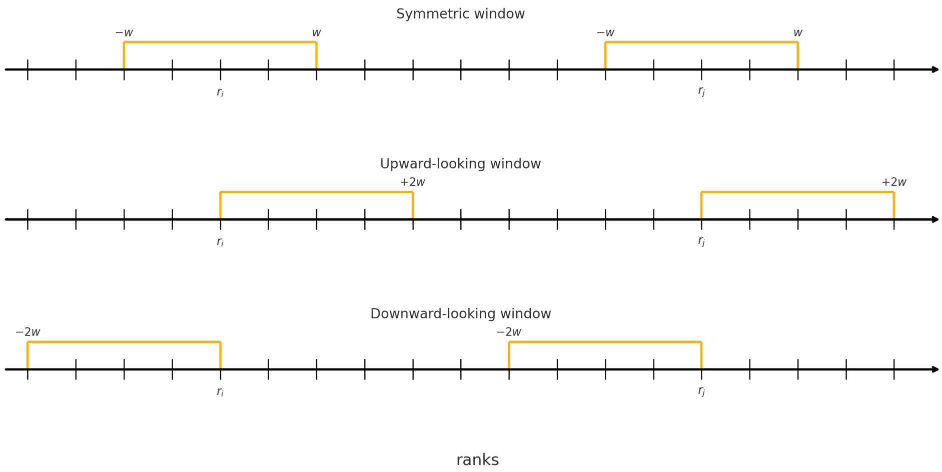

Figure 1: Symmetric, upward-looking and downward-looking windows around ranks r_i and r_j. Upward/downward windows span $2w$ ranks above/below the focal rank.

Under symmetric sampling, exactly w observed ranks lie below r and w lie above, so $s(r) = w + 1$ and

$$\hat{p}^{\mathrm{sym}}(r) = \frac{1}{2}. \tag{7}$$

Given a window $\mathcal{W}(r)$ of size N, define the local mean income

$$\bar{y}_r = \frac{1}{N} \sum_{k \in \mathcal{W}(r)} y_{(k)}. \tag{8}$$

The local Gini at rank r is

$$\tilde{G}(r) = \frac{1}{2N^2 \bar{y}_r} \sum_{k \in \mathcal{W}(r)} \sum_{\ell \in \mathcal{W}(r)} \left| y_{(k)} - y_{(\ell)} \right|. \tag{9}$$

A rank-correlation representation, following Milanovic (1997) and related covariance formulations (Lerman and Yitzhaki, 1984), is

$$\tilde{G}(r) = 2\, \frac{\mathrm{Cov}(Y, s)}{N \bar{y}_r}, \tag{10}$$

where $s \in \{1, \ldots, N\}$ are within-window ranks from poorest to richest and Y is the within-window income random variable assigning mass $1/N$ to each observed income.

Define the discrete window-width factor

$$c_w = \frac{2w(w+1)}{2w+1}. \tag{11}$$

Linear Pen's parade

Consider a curvature-free baseline where income is linear in rank,

$$y_{(r)} = br, \qquad b > 0. \tag{12}$$

In the continuous limit, a linear parade corresponds to a uniform distribution on an interval, and Milanovic (1997) derives that the associated population Gini equals

$$G^{\mathrm{lin}} = \frac{1}{3}. \tag{13}$$

Proposition 1 (Linear parade: exact local Gini, strict underestimation, and monotonicity)**.** Under (12), the symmetric-window local Gini is

$$\tilde{G}^{\text{sym}}(r) = \frac{2}{3}\,\frac{w(w+1)}{(2w+1)\,r}. \tag{14}$$

For all interior ranks $r \in \{w+1, \ldots, n-w\}$,

$$\tilde{G}^{\text{sym}}(r) < G^{\text{lin}} = \frac{1}{3}. \tag{15}$$

Moreover, $\tilde{G}^{\text{sym}}(r)$ is strictly decreasing in r on the interior range.

Proof. The exact expression (14) follows from (10) because within-window income is an affine function of within-window rank, yielding correlation one and a within-window variance that depends only on $N = 2w+1$. For the inequality, use $r \geq w+1$:

$$\begin{aligned}\tilde{G}^{\text{sym}}(r) &\leq \tilde{G}^{\text{sym}}(w+1)\\ &= \frac{2}{3}\frac{w(w+1)}{(2w+1)(w+1)}\\ &= \frac{2}{3}\frac{w}{2w+1}\\ &< \frac{1}{3}.\end{aligned} \tag{16}$$

Monotonicity is immediate from (14) because it is proportional to $1/r$. □

Corollary 1 (Linear parade: directional windows also underestimate)**.** Under (12), the directional local Ginis satisfy

$$\tilde{G}^{\text{up}}(r) = \frac{2}{3}\,\frac{w(w+1)}{(2w+1)(r+w)}, \qquad \tilde{G}^{\text{down}}(r) = \frac{2}{3}\,\frac{w(w+1)}{(2w+1)(r-w)}. \tag{17}$$

On their interior rank ranges, both satisfy $\tilde{G}(r) < 1/3$.

Curved parades: first-order approximation framework

For curved parades, local income spacings vary with rank, and closed-form expressions for the discrete statistic (9) are generally not available. The results below therefore use a first-order quantile approximation that yields explicit rank-dependent formulas. The theorems in the exponential and Pareto sections are stated for these approximation formulas, and the accompanying figures provide numerical validation that the discrete sliding-window Gini behaves analogously for the finite (n, w) used in the simulations.

Let $Q(p)$ denote the population quantile function and set

$$p = \frac{r}{n+1}. \tag{18}$$

A first-order expansion of Q around p implies that, within a rank window around r, incomes are locally approximated by an affine function of rank. This local linearization delivers explicit approximation formulas for the local Gini in the exponential and Pareto regimes below.

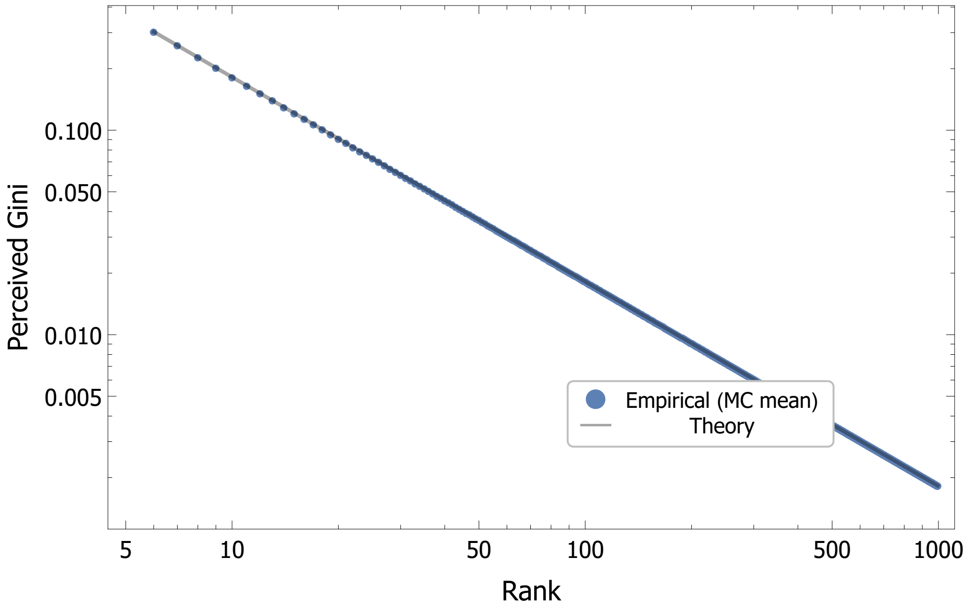

Figure 2: Local Gini coefficients under a linear parade using $w = 5$. Discrete points show Monte Carlo mean (50 MC iterations) local Ginis. The superposed points show the exact expression in (14). Double-logarithmic scale.

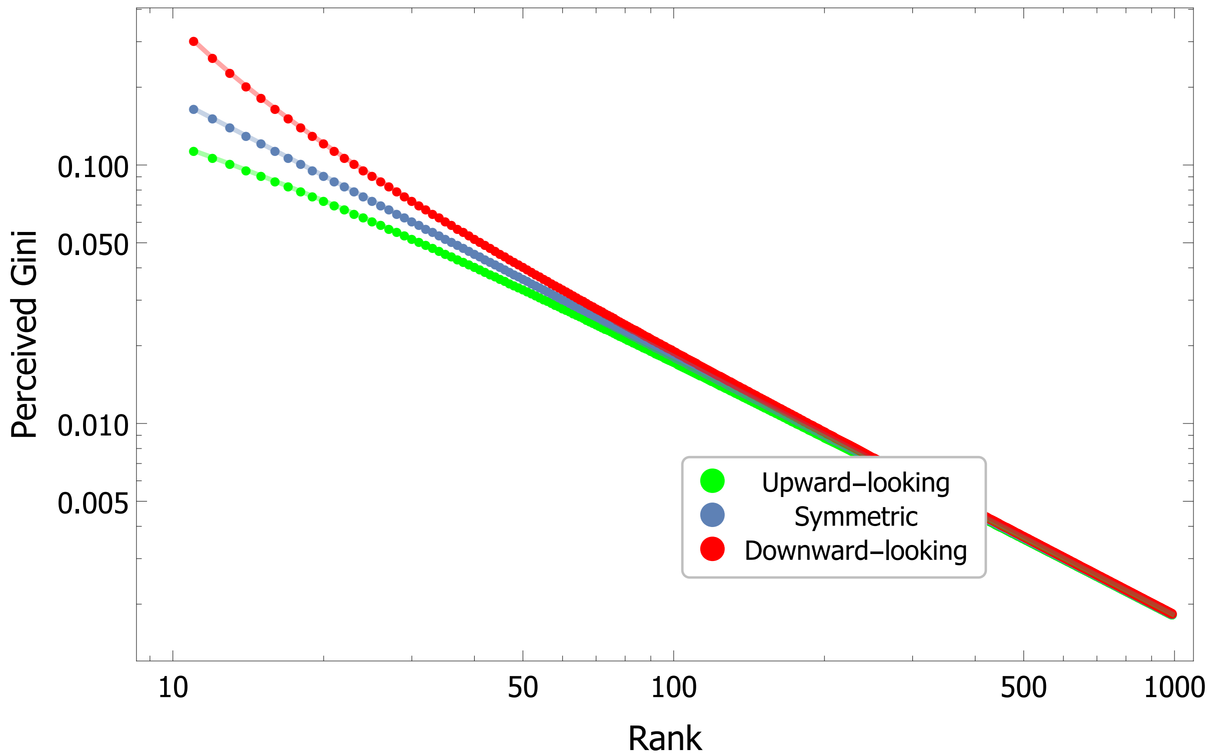

Figure 3: Local Gini coefficients under symmetric, upward-looking, and downward-looking windows using $w = 5$. Discrete points show Monte Carlo mean local Ginis (50 MC iterations) and superposed points show the exact expressions in (14) and (17). Double-logarithmic scale.

Exponential regime

Exponential regimes are empirically relevant in the bulk of income distributions in exponential–power-law mixtures (Silva and Yakovenko, 2005). For an exponential distribution with parameter $\lambda > 0$, the population Gini equals

$$G^{\exp} = \frac{1}{2}. \tag{19}$$

Using a first-order quantile expansion yields the explicit approximation used in the superposed theory points:

$$G^{\exp}_{\mathrm{FO}}(r) \approx \frac{w}{3\,(n+1-r)\,\left[-\ln\!\left(\frac{n+1-r}{n+1}\right)\right]}. \tag{20}$$

Proposition 2 (Exponential regime: strict underestimation for the first-order approximation)**.** Let $G^{\exp}_{\mathrm{FO}}(r)$ be defined by (20) on the interior rank range $r \in \{w+1, \dots, n-w\}$. Then

$$G^{\exp}_{\mathrm{FO}}(r) < \frac{1}{2} = G^{\exp} \tag{21}$$

for all such r.

Proof. Let $A = n+1$ and $x = A-r$. On the interior rank range, $x \in \{w+1, \dots, A-w-1\}$. Write (20) as

$$G^{\exp}_{\mathrm{FO}}(r) = \frac{w}{3h(x)}, \qquad h(x) = x\ln\!\left(\frac{A}{x}\right). \tag{22}$$

Since

$$h''(x) = -\frac{1}{x} < 0, \tag{23}$$

h is strictly concave, so its minimum on $[w+1, A-w-1]$ is attained at an endpoint.

At the lower endpoint,

$$h(w+1) = (w+1)\ln\!\left(\frac{A}{w+1}\right) \geq (w+1)\ln 2, \tag{24}$$

because $A = n+1 \geq 2w+2$ on the interior range.

For the upper endpoint, let $c = w+1$ and $u = A/c \geq 2$. Then

$$h(A-c) = c(u-1)\ln\!\left(\frac{u}{u-1}\right). \tag{25}$$

Define $\phi(u) = (u-1)\ln\!\big(u/(u-1)\big)$ for $u \geq 2$. Differentiating gives

$$\phi'(u) = \ln\!\left(\frac{u}{u-1}\right) - \frac{1}{u} > 0, \tag{26}$$

so ϕ is increasing on $[2, \infty)$. Therefore,

$$h(A-c) = c\phi(u) \geq c\phi(2) = c\ln 2 = (w+1)\ln 2. \tag{27}$$

Hence, for all interior ranks,

$$h(x) \geq (w+1)\ln 2. \tag{28}$$

Since $\ln 2 > 2/3$, it follows that

$$h(x) > (w+1)\frac{2}{3} > \frac{2w}{3}. \tag{29}$$

Substituting into (22) yields

$$\begin{aligned} G_{\mathrm{FO}}^{\exp}(r) &= \frac{w}{3h(x)} \\ &< \frac{w}{3(2w/3)} \\ &= \frac{1}{2} \\ &= G^{\exp}. \end{aligned} \tag{30}$$

□

Proposition 3 (Exponential regime: turning point and U-shaped rank profile for the first-order approximation)**.** Let $G_{\mathrm{FO}}^{\exp}(r)$ be defined by (20) on the interior rank range $r \in \{w+1, \ldots, n-w\}$. Define $A = n+1$ and $x = A - r$. Then $G_{\mathrm{FO}}^{\exp}(r)$ is decreasing in r for $r < r^*$ and increasing in r for $r > r^*$, where

$$r^* = A - \frac{A}{e} = (n+1)\left(1 - \frac{1}{e}\right). \tag{31}$$

Proof. Write (20) as $G_{\mathrm{FO}}^{\exp}(r) = w/(3f(r))$ with

$$f(r) = (A-r)\left[-\ln\left(\frac{A-r}{A}\right)\right]. \tag{32}$$

Let $x = A - r$ so that $f(r) = x\ln(A/x)$. Differentiating yields

$$\frac{d}{dx}\left[x\ln\left(\frac{A}{x}\right)\right] = \ln\left(\frac{A}{x}\right) - 1. \tag{33}$$

The derivative is zero at $x = A/e$. For $x > A/e$, it is positive, so f increases with x and therefore decreases with r, implying that $G_{\mathrm{FO}}^{\exp}(r)$ decreases with r. For $x < A/e$, it is negative, so f decreases with x and therefore increases with r, implying that $G_{\mathrm{FO}}^{\exp}(r)$ increases with r. Translating $x = A/e$ back to ranks yields (31). □

Pareto regime

Power-law regimes characterize upper tails in exponential–power-law mixtures (Dragulescu and Yakovenko, 2001; Silva and Yakovenko, 2005). For a Pareto type-I distribution with tail index $\alpha > 1$, the population Gini equals

$$G^{\mathrm{Par}} = \frac{1}{2\alpha - 1}. \tag{34}$$

A first-order quantile expansion combined with the discrete correction factor c_w in (11) yields the explicit approximation

$$G_{\mathrm{FO}}^{\mathrm{Par}}(r) \approx \frac{c_w}{3\alpha(n+1-r)} = \frac{2w(w+1)}{3(2w+1)\,\alpha\,(n+1-r)}. \tag{35}$$

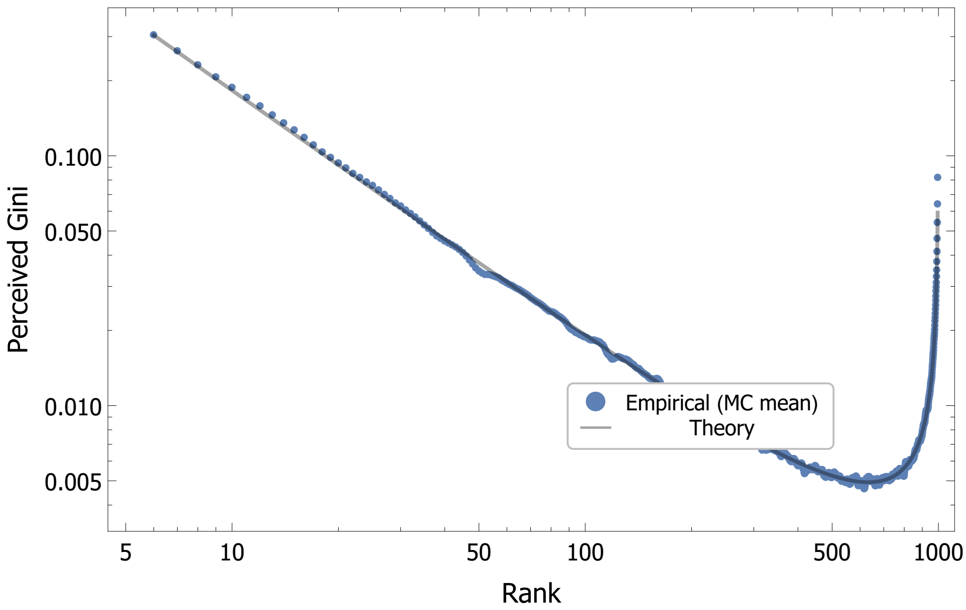

Figure 4: Local Gini coefficients under an exponential regime using $w = 5$. Discrete points show Monte Carlo mean local Ginis (50 MC iterations) and superposed points show the explicit first-order approximation in (20). Double-logarithmic scale.

Proposition 4 (Pareto regime: strict underestimation and monotonicity for the first-order approximation)**.** Assume $\alpha > 1$ for the mean to exist and let $G^{\mathrm{Par}}_{\mathrm{FO}}(r)$ be defined by (35) on the interior rank range $r \in \{w+1, \dots, n-w\}$. Then

$$G^{\mathrm{Par}}_{\mathrm{FO}}(r) < G^{\mathrm{Par}} = \frac{1}{2\alpha - 1} \tag{36}$$

for all such r. Moreover, $G^{\mathrm{Par}}_{\mathrm{FO}}(r)$ is strictly increasing in r on the interior rank range.

Proof. Since $n + 1 - r \geq w + 1$ on the interior range,

$$\begin{aligned} G^{\mathrm{Par}}_{\mathrm{FO}}(r) &\leq \frac{2w(w+1)}{3(2w+1)\alpha(w+1)} \\ &= \frac{2w}{3(2w+1)\alpha} \\ &< \frac{1}{3\alpha}. \end{aligned} \tag{37}$$

For $\alpha > 1$, one has $1/(3\alpha) \leq 1/(2\alpha - 1)$ because $2\alpha - 1 \leq 3\alpha$ is equivalent to $\alpha \geq -1$. Hence $G^{\mathrm{Par}}_{\mathrm{FO}}(r) < 1/(2\alpha - 1) = G^{\mathrm{Par}}$. Monotonicity is immediate from (35) because it is proportional to $1/(n + 1 - r)$, which increases with r. $\square$

Corollary 2 (Pareto–linear comparison: local profile crossing and misranking)**.** Let $A = n+1$ and compare the exact linear symmetric-window local Gini (14) to the Pareto first-order approximation (35) on their common interior rank range. The two profiles cross at

$$r^{\times} = \frac{\alpha A}{1 + \alpha} = \frac{\alpha(n+1)}{1 + \alpha}. \tag{38}$$

More precisely,

$$G^{\mathrm{Par}}_{\mathrm{FO}}(r) < \tilde{G}^{\mathrm{sym}}(r) \quad \Longleftrightarrow \quad r < r^{\times}. \tag{39}$$

If $1 < \alpha < 2$, then $G^{\mathrm{Par}} > G^{\mathrm{lin}} = 1/3$ globally, while all ranks satisfying (39) locally rank the Pareto society as less unequal than the linear society.

Proof. Using (14) and (35),

$$G^{\mathrm{Par}}_{\mathrm{FO}}(r) - \tilde{G}^{\mathrm{sym}}(r) = \frac{c_w}{3}\left[\frac{1}{\alpha(A-r)} - \frac{1}{r}\right] = \frac{c_w\big((1+\alpha)r - \alpha A\big)}{3\alpha r(A-r)}. \tag{40}$$

On the interior range, the denominator is positive, so the sign is determined by $(1+\alpha)r - \alpha A$, which yields (38) and (39). For the global ordering, (34) and (13) imply

$$G^{\mathrm{Par}} > \frac{1}{3} \quad \Longleftrightarrow \quad \frac{1}{2\alpha-1} > \frac{1}{3} \quad \Longleftrightarrow \quad \alpha < 2. \tag{41}$$

Combined with $\alpha > 1$, this gives the stated misranking region for $1 < \alpha < 2$. □

Corollary 3 (Pareto–linear comparison: majority misranking for $1 < \alpha < 2$ (continuous-rank sense))**.** In the continuous-rank extension of Corollary 2, if $1 < \alpha < 2$, then a strict majority of ranks misrank the Pareto society as less unequal than the linear society:

$$\lambda\Big(\Big\{r \in (0, n+1) : G^{\mathrm{Par}}_{\mathrm{FO}}(r) < \tilde{G}^{\mathrm{sym}}(r)\Big\}\Big) > \frac{n+1}{2}, \tag{42}$$

where $\lambda(\cdot)$ denotes interval length.

Proof. By Corollary 2, the misranking set is $(0, r^{\times})$ in the continuous-rank extension. Since $\alpha > 1$,

$$r^{\times} = \frac{\alpha(n+1)}{1+\alpha} > \frac{n+1}{2} \quad \Longleftrightarrow \quad \frac{\alpha}{1+\alpha} > \frac{1}{2}, \tag{43}$$

which holds if and only if $\alpha > 1$. For $1 < \alpha < 2$, the global ordering is Pareto above linear by Corollary 2, so a strict majority misranks. In finite n, integer rounding and boundary exclusions can generate knife-edge cases near $\alpha \downarrow 1$, especially for even n, but the majority statement holds asymptotically and in the continuous-rank sense. □

Proposition 5 (Pareto regime: underreaction of locally perceived inequality to changes in actual inequality)**.** Fix an interior rank r and let $m_r = n + 1 - r$. Under the Pareto first-order approximation (35), locally perceived inequality can be written as a function of the population Pareto Gini (34):

$$G^{\mathrm{Par}}_{\mathrm{FO}}(r) = \frac{2c_w}{3m_r}\,\frac{G^{\mathrm{Par}}}{1+G^{\mathrm{Par}}}. \tag{44}$$

Hence, for fixed r and w,

$$\frac{\partial G^{\mathrm{Par}}_{\mathrm{FO}}(r)}{\partial G^{\mathrm{Par}}} = \frac{2c_w}{3m_r(1+G^{\mathrm{Par}})^2} < 1, \tag{45}$$

and in fact

$$\frac{\partial G^{\mathrm{Par}}_{\mathrm{FO}}(r)}{\partial G^{\mathrm{Par}}} < \frac{2c_w}{3m_r} \le \frac{4w}{3(2w+1)} < \frac{2}{3}. \tag{46}$$

The elasticity of locally perceived inequality with respect to actual inequality is

$$\frac{\partial \ln G^{\mathrm{Par}}_{\mathrm{FO}}(r)}{\partial \ln G^{\mathrm{Par}}} = \frac{1}{1+G^{\mathrm{Par}}} = \frac{2\alpha-1}{2\alpha} \in \left(\frac{1}{2}, 1\right). \tag{47}$$

Proof. From (34), one has $\alpha = (1 + 1/G^{\mathrm{Par}})/2$. Substituting into (35) yields (44). Differentiating gives (45). Since $(1 + G^{\mathrm{Par}})^2 > 1$, the derivative is strictly smaller than $2c_w/(3m_r)$. On the interior range, $m_r \geq w + 1$, so

$$
\begin{aligned}
\frac{2c_w}{3m_r} &\leq \frac{2}{3}\frac{2w(w+1)}{(2w+1)(w+1)} \\
&= \frac{4w}{3(2w+1)} \\
&< \frac{2}{3},
\end{aligned}
\tag{48}
$$

which proves (46). For the elasticity, differentiate (44) logarithmically:

$$
\begin{aligned}
\frac{\partial \ln G^{\mathrm{Par}}_{\mathrm{FO}}(r)}{\partial \ln G^{\mathrm{Par}}} &= \frac{\partial}{\partial \ln G^{\mathrm{Par}}}\left[\ln G^{\mathrm{Par}} - \ln(1 + G^{\mathrm{Par}})\right] \\
&= 1 - \frac{G^{\mathrm{Par}}}{1 + G^{\mathrm{Par}}} \\
&= \frac{1}{1 + G^{\mathrm{Par}}}.
\end{aligned}
\tag{49}
$$

Using (34) gives (47). Since $\alpha > 1$, one has $0 < G^{\mathrm{Par}} < 1$, so the elasticity lies in $(1/2, 1)$. $\square$

A useful implication of Proposition 5 is that, within the Pareto family, increases in actual inequality (equivalently, decreases in α) raise locally perceived inequality in the same direction but less strongly than one-for-one. In particular, the local-to-global ratio

$$
\frac{G^{\mathrm{Par}}_{\mathrm{FO}}(r)}{G^{\mathrm{Par}}} = \frac{c_w(2\alpha - 1)}{3\alpha(n + 1 - r)}
\tag{50}
$$

decreases as α falls, so proportional underestimation becomes more severe as the Pareto tail thickens.

3 Conclusion

This paper develops a local inequality statistic for socially constrained observability. By computing a Gini coefficient on ego-centered rank windows along Pen's parade, it isolates a measurement channel through which local information alone generates systematic heterogeneity in perceived inequality across ranks. Symmetric self-centered sampling also implies a mechanical central self-placement in the local window, $\hat{p}^{\mathrm{sym}}(r) = 1/2$, independently of objective rank. If agents infer broader social position from such local comparison sets and extrapolate beyond them, this mechanism is consistent with observed middle-class self-placement.

The main analytical contribution is a family of strict local–global inequalities and simple rank profiles. In the linear parade, the local Gini admits an exact closed form (14), strict underestimation relative to the population benchmark $G^{\mathrm{lin}} = 1/3$ holds uniformly on the interior rank range, and perceived inequality decreases hyperbolically in rank (Proposition 1). In curved parades, the analytical results are stated for first-order approximation formulas. For an exponential regime, the first-order approximation is

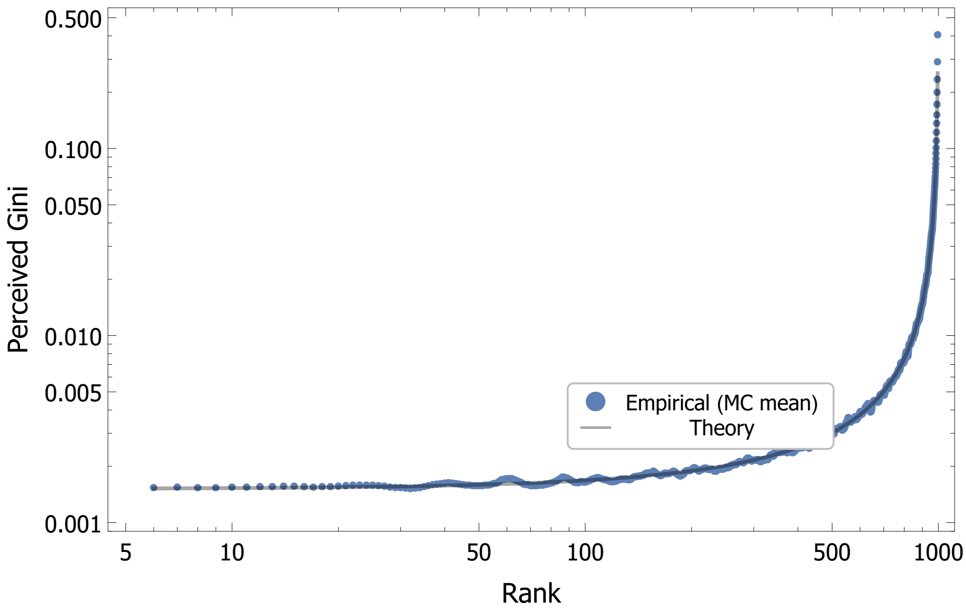

Figure 5: Local Gini coefficients under a Pareto regime using $w = 5$. Discrete points show Monte Carlo mean local Ginis (50 MC iterations) and superposed points show the explicit first-order approximation in (35). Double-logarithmic scale.

strictly below the exponential population Gini 1/2 on the interior rank range (Proposition 2) and exhibits a turning point at $r^* = (n+1)(1-1/e)$, yielding a U-shaped rank profile (Proposition 3). For a Pareto regime, the first-order approximation (35) is uniformly bounded above by a term strictly below the Pareto population Gini $1/(2\alpha-1)$ and is monotonously increasing in rank on the interior range (Proposition 4). The figures provide numerical validation that the discrete sliding-window Gini computed from finite samples behaves analogously for the Monte Carlo designs used here.

Beyond underestimation, the benchmark comparisons show that local perceived inequality need not preserve the cross-society ordering induced by population Ginis. In the Pareto–linear comparison, the local profiles cross at a rank cutoff $r^\times = \alpha(n+1)/(1+\alpha)$ (Corollary 2). For $1 < \alpha < 2$, the Pareto society is globally more unequal than the linear benchmark, yet all ranks below this cutoff locally rank it as less unequal; in the continuous-rank extension, this implies majority misranking (Corollary 3). Within the Pareto family, locally perceived inequality responds to increases in actual inequality in the same direction but less than one-for-one in levels and with elasticity strictly below one (Proposition 5), so proportional underestimation becomes more severe as tails thicken.

The magnitude of the local Gini is governed by the ratio of window width to rank scale, so for fixed w and large populations the statistic can become very small for much of the interior rank range. This should be interpreted as a property of inequality within a locally observed stratum, not as a literal claim that agents necessarily perceive society-wide equality. At the same time, if agents form beliefs about aggregate inequality by extrapolating from such social samples, the resulting local statistic provides a natural measurement channel through which underestimation and rank-dependent heterogeneity in beliefs may arise.

The central implication is not only local underestimation, but non-preservation of inequality orderings under local observability. Once inequality is measured on socially sampled comparison sets, parade curvature and tail shape can induce profile crossings and rank-majority reversals relative to population-level comparisons. In this sense, the framework identifies a structural source of disagreement in inequality perceptions that arises at the level of observability and measurement, before any additional behavioral assumptions are imposed.

The paper isolates the consequences of rank-adjacent observability by construction: comparison sets are local windows on Pen's parade rather than generic samples of size N. Accordingly, the results characterize one specific mechanism, homophilic rank-local sampling, and its implications for local inequality measurement. Natural extensions would study alternative sampling strategies and compare the resulting local inequality profiles, including random sampling, stratified sampling, or other network-based sampling rules.

Statements and Declarations

Competing interests

The author has no competing interests to declare.

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